Relativity Module
Student Guide

Concepts of this Module

- Events that can cause other events
- Synchronising clocks
- Simultaneity
- Time dilation and inertial reference frames
- Rigid bodies and relativity
- Tachyons
- Geometric approaches to relativity
- Photons and relativity
- Mass-energy equivalence
- The Equivalence Principle
- Geometry

The Activities

Course Concepts *Activity 1*

Event 1 occurs at $x = 0$ and $t = 0$.

A. Event 2 occurs at x = 1200 *m* and is caused by Event 1. When is the earliest that Event 2 can occur. Explain.
B. Event 0 occurred at x = -4,000 *m* at time t = -1.0 μs. Could Event 0 have caused Event 1? Explain.

Activities 2 and 3 refer to the following situation.

Assume Toronto and Montreal are exactly 510 km apart in the Earth frame of reference. Assume that Kingston is exactly half way between Toronto and Montreal. They are all, of course, stationary relative to each other and the Earth. Ignore any effects due to the Earth's rotation on its axis. Ignore any effects due to the Earth's gravitational field. Assume the surface of the Earth is flat. Assume that the speed of light in the air is exactly equal to c.

A powerful searchlight is on top of the CN Tower in Toronto, pointed towards Montreal. A second searchlight is on top of 1000 de La Gauchetière in Montreal, pointed towards Toronto. Initially the two searchlights are turned off. Assume that both the CN Tower and 1000 de La Gauchetière are visible from each other and from any point between Toronto and Montreal.

There are also two identical atomic clocks with an accuracy of at least 1 μs, one on top of the CN Tower and the other on top of 1000 de La Gauchetière.

Course Concepts *Activity 2*

A. You are stationary in Kingston. With two powerful telescopes you are looking at the two clocks. You see that they read the same time. Are the clocks synchronized? If yes, explain. If no, which clock is ahead and by how much?
B. You are on top of the CN Tower right beside its clock. With a powerful telescope you look at the clock in Montréal, and see that it reads exactly the same time you see on the clock on the CN Tower. Are the clocks synchronized? If yes, explain. If no, which clock is ahead and by how much?
C. You are on top of 1000 de La Gauchetière right beside its clock and see that it reads a time of 12.000000 s. With a powerful telescope you simultaneously look at the clock in Toronto. If the two clocks are synchronized, what time should you see the Toronto clock read?

Course Concepts *Activity 3*

A. You are in a rocket ship traveling from Toronto to Montreal at 0.8 c relative to the Earth. For you what is the distance between Toronto and Montreal? For you what is the distance between Toronto and Kingston?
B. The two searchlights on the CN Tower in Toronto and 1000 de La Gauchetière in Montreal are quickly turned on and off, both emitting flashes of light. **For you, the two flashes were emitted simultaneously.** Imagine you are right beside the CN Tower when its light is turned on. You see the flash from the searchlight on the CN Tower instantaneously. Sketch the two flashes of light, from the CN Tower and from 1000 de La Gauchetière, a very brief moment after they are emitted, indicating their speeds and the distance between them for you. How long before you see the flash from the searchlight on 1000 de La Gauchetière? Be sure to clearly indicate how you arrived at your answer.
C. Another member of your Team also is in a rocket flying at 0.8 c from Toronto to Montreal. What is your Teammate's speed relative to you? Imagine your Teammate is just over Kingston when the flashes are emitted. Are the two flashes of light emitted simultaneously for your Teammate? Will he/she see the flashes

simultaneously? If yes, how long after the flashes will he/she see them? If no, which flash will he/she see first and by how much? You may find it useful to add your Teammate to your sketch from Part B.

D. Your Instructor has gone to Kingston, and is stationary relative to Kingston and the Earth. What is your Instructor's speed relative to you? Will your Instructor see the flashes from the two searchlights of Part B simultaneously? If no, which flash will he/she see first and by how much as measured by you? Explain. You may find it useful to add your Instructor to the sketch from Parts B and C.
E. Imagine your Instructor has a cassette player that will begin playing when it receives a flash of light, and quits when it receives a second flash of light. Is there any music? Does the tape move? Do you hear any music? Does your Instructor hear any music?

Course Concepts *Activity 4*

George and Helen are twins, born at the same time (a biological impossibility). George stays at home on Earth, which we assume is a good inertial reference frame. Helen is an astronaut who blasts off for a distant star, travels to the star at high speed, and then turns around and returns to Earth. After she lands on Earth she will be younger than George

A. Draw a spacetime diagram for a reference frame stationary relative to George. Include George's worldline and Helen's worldline. What can you conclude about the relation between the length of the two worldlines and which twin ends up younger?
B. Draw a spacetime diagram for an inertial reference frame in which Helen is stationary during her trip from Earth to the distant star. Include George's worldline and Helen's worldline. What can you conclude about the relation between the length of the two worldlines and which twin ends up younger? Is this consistent with your conclusion from Part A?
C. Do you think this is a general result: that when analysed from *any* inertial reference frame Helen's worldline is longer than George's, and she will end up younger than George?

Course Concepts *Activity 5*

In 1971 Hafele and Keating tested the predictions of the Theories of Relativity by flying cesium beam atomic clocks around the world on regularly scheduled commercial airline flights. One clock remained in their laboratory outside Washington DC, one was flown to the East, and the other to the West. They got the data on speeds and altitudes of the planes flying the Eastbound and Westbound clocks from the flight recorders.

When the got all three clocks back in the laboratory they compared the measured times. Here are the predicted and experimental results for the elapsed times compared to the clock that stayed in the laboratory.

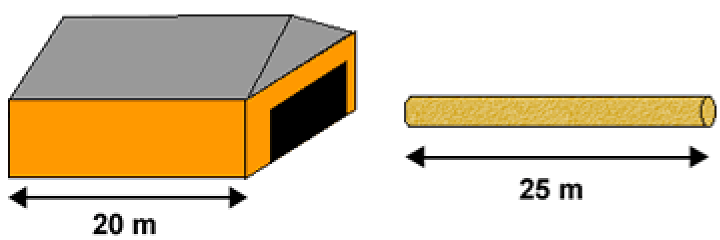
	Eastbound clock (ns)	Westbound clock (ns)
Special Relativity prediction	lose 184 ± 18	gain 96 ± 10
General Relativity prediction	gain 144 ± 14	gain 179 ± 18
Total predicted effect	lose 40 ± 23	gain 275 ± 21
Measured	lost 59 ± 10	gained 273 ± 7

The Special Relativity prediction is because that theory predicts that moving clocks run slowly. The General Relativity prediction is because that theory predicts that clocks in gravitational fields run slowly.

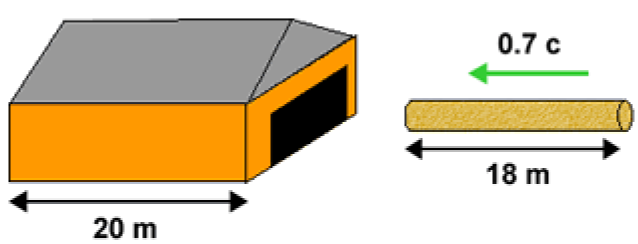
A. Why does Special Relativity predict that the Eastbound clock gains time and the Westbound clock loses time? Shouldn't they both either gain time or lose time?
B. Do the predicted effects agree with the experimental data? Explain.
C. Are the calculated errors in the total predicted effect correct? How were these errors calculated?

Course Concepts *Activity 6*

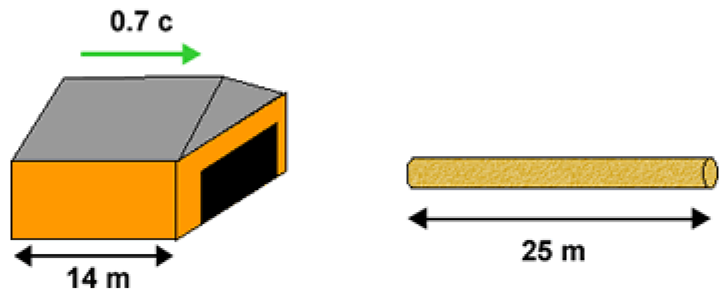
We have a 25 m long pole and a 20 m long barn, both as measured at rest relative to the pole and the barn. We will assume the back wall of the barn is very very strong.



If the pole is moving towards the barn at 70% of the speed of light, its length will be contracted to about 18 m. Thus it clearly fits in the barn, and we can slam the door shut (and run!).



But if we are riding along with the pole, its length is not contracted and is 25 m long. But the barn is contracted and is now about 14 m long. Clearly the pole does not fit in the barn.



Does the pole fit into the barn or not? Explain.

Course Concepts *Activity 7*

For a long time people interpreted the Special Theory of Relativity to mean that *nothing* can travel faster than the speed of light. In 1967 Feinberg showed that this is not correct. There is room in the theory for objects whose speed is always greater than *c*. Feinberg called these hypothetical objects *tachyons*; the word has the same root as, say, tachometer. Attempts have been made to observe tachyons: so far all such experiments have failed.

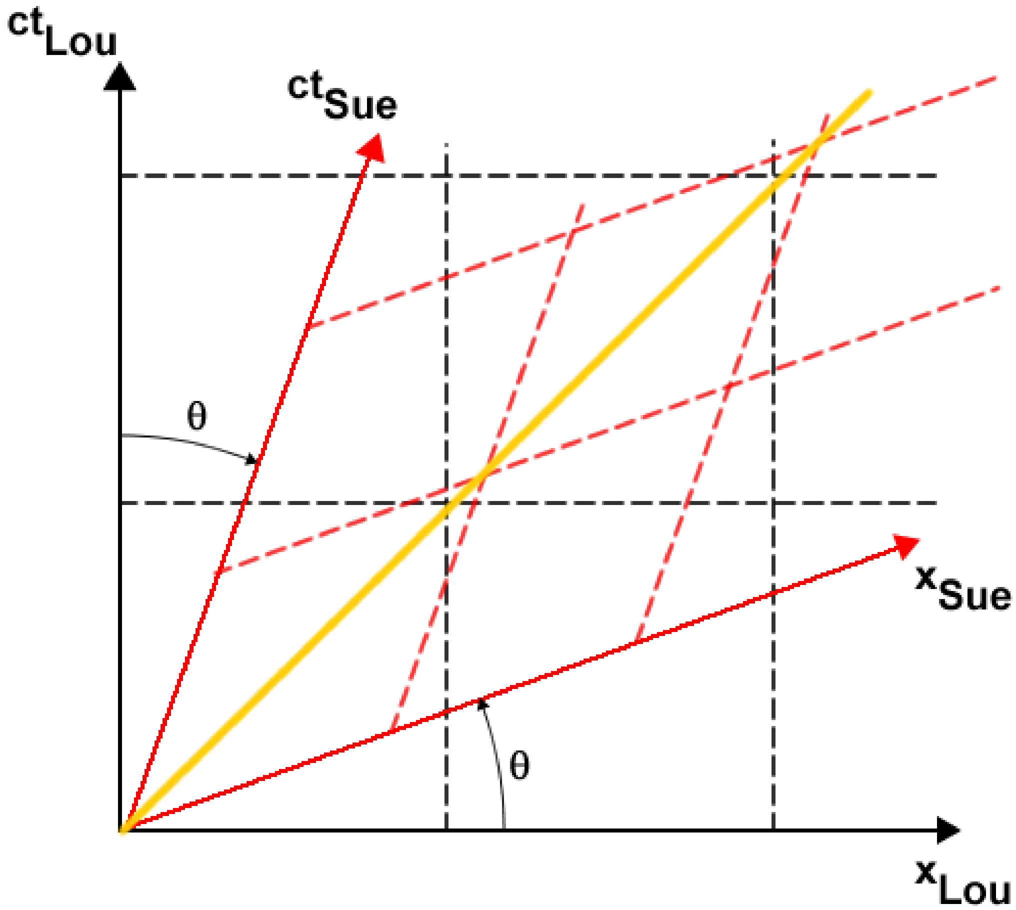
A. The relation between energy *E* and mass *m* for an object traveling at speed *u* relative to us is given by:

   $$E = \frac{mc^2}{\sqrt{1-u^2/c^2}}$$

   If the energy of a tachyon is a real number, what kind of number must its mass *m* be?
B. Draw energy versus speed axes with the speed going from 0 to 5 c. Sketch the relation between the energy *E* and speed *u* for an ordinary object. On the same plot sketch the relation between *E* and *u* for a tachyon. For ordinary objects we say: "It takes infinite energy to speed the object up to a speed equal to the speed of light." What is the equivalent statement for a tachyon? For ordinary objects we can also say: "The minimum value of the energy is when it is at rest, and has a value equal to mc^2." What is the equivalent statement for a tachyon?
C. A tachyon is produced by some apparatus in the laboratory at $x = 0$ at $t = 0$ and travels at $u_x = 20\,c$ in the +x direction to a detector at $x = 1{,}000\ m$. You are traveling at $v = 0.1\ c$ in the +x direction relative to the laboratory. The formula for addition of velocities is: $u_x' = \frac{u_x - v}{1 - u_x v/c^2}$. What is the speed of the tachyon relative to you?
D. For Part C describe in words what you see the tachyon doing. What does this say about the tachyon being produced by some apparatus causing its later detection by the detector.

Course Concepts *Activity 8*

Sue and Lou are moving relative to each other at speed *v*. We can relate their spacetime diagrams as shown.



In the above diagram:

$$\tan(\theta) = \frac{v}{c}$$

Similar to the Parable of the Surveyors, the axes are rotated relative to each other, but they are rotating in opposite directions. Also shown in yellow is the worldline for light traveling at *c* relative to both Sue and Lou.

You will be using this diagram below, and may find it useful to print a few copies of it to staple into your lab book.

A. Explain how the diagram shows that the speed of light is the same value for both Sue and Lou.
B. Place two dots on the diagram representing the positions and times of two events that are simultaneous for Lou. Are the two events simultaneous for Sue? If no, which event occurs first? Place two more dots on the diagram representing the

positions and times of two events that are simultaneous for Sue. Are the two events simultaneous for Lou? If no, which event occurs first?

C. Sketch the worldline of an object that is stationary relative to Lou. What is the direction of motion of the object relative to Sue? Explain.
D. Sketch the worldline of an object moving at some speed $u < c$ relative to Lou. Show geometrically that the object is moving at speed $u' < c$ relative to Sue.
E. Activity 6 introduced *tachyons*, objects whose speed is always greater than the speed of light. Draw the worldline of a tachyon moving at some speed $u > c$ relative to Lou. Show geometrically that the direction of motion of the tachyon for Sue, u', can be negative but with a magnitude $> c$.

Course Concepts *Activity 9*

As you may know, in some circumstances we can treat light as a particle called a *photon*.

A. Relativistic time dilation means that an unstable particle which decays in time $\Delta\tau$ when it is rest relative to some observer will live a longer time Δt for an observer for whom it is moving with speed v where:

$$\Delta t = \frac{\Delta\tau}{\sqrt{1 - v^2/c^2}} > \Delta\tau$$

If you apply this formalism to a photon, what does it predict about its lifetime relative to any observer?

B. The relation between energy E and mass m for an object traveling at speed u relative to us is given by:

$$E = \frac{mc^2}{\sqrt{1 - u^2/c^2}}$$

A photon has a real non-zero energy. What does this equation say about the value of its mass m? What do you think your high school math teacher would say about your answer?

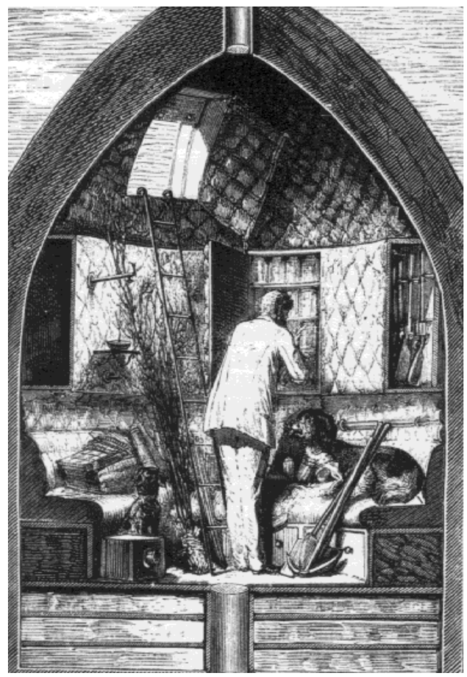
Activity 10

Four elementary particles, an electron, a muon, a proton, and a neutron have the rest energies and relativistic total energies as shown.

Particle	Rest energy	Total energy
Electron	0.511 MeV	0.511 Mev
Muon	106MeV	212 MeV
Proton	938 MeV	4,690 MeV
Neutron	940 MeV	2,820 MeV

Rank in order from the largest to the smallest the particle's speeds.

Activity 11



In Jules Verne's **From the Earth to the Moon** (1865) a huge cannon fires a projectile at the moon. Inside the projectile was furniture, three people and two dogs. The figure is from the original edition.

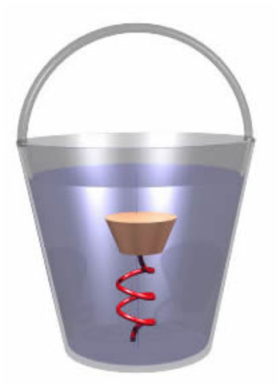
Verne reasoned that at least until the projectile got close to the Moon it would be in the Earth's gravitational field during its journey. Thus the people and dogs would experience normal gravity, and be able to, for example, sit on the chairs just as if the projectile were sitting on the Earth's surface.

One of the dogs died during the trip. They put the dog's body out the hatch and into space. The next day the people looked out the porthole and saw that the dog's body was still floating just beside the projectile.

A. Is there a contradiction between the inhabitants inside the projectile experiencing normal gravity and the dog's body outside the projectile not falling back to the Earth?
B. If your answer to Part A is yes, where did Verne make his mistake? If your answer is no, explain.

Activity 12

A bucket of water has a spring soldered to the bottom, as shown. A cork is attached the spring, and is therefore suspended under the surface of the water.



You are on top of the CN tower, holding the bucket, and step off. While falling towards the ground, do you see the cork move towards the top of the water, towards the bottom of the bucket, or stay where it is relative to the bucket and the water?

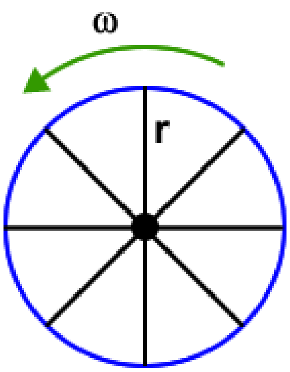
Explain your answer using Einstein's Equivalence Principle.

Activity 13

As you know, according to plane geometry the circumference C_0 of a circle is related to its radius r according to:

$$C_0 = 2\pi r$$

Imagine we have a wheel of radius r whose circumference when it is stationary is C_0. But the wheel is rotating with constant angular velocity ω. According to Special Relativity, the rim of the wheel will be contracted. Assume the rim has negligible thickness.



A. Will the length of the spokes of the wheel be contracted too? Explain.
B. What is the circumference C of the wheel according to Special Relativity? Express your answer in terms of the centripetal (radial) acceleration a of each point on the rim.
C. Following Einstein, we say that the geometry of the rotating wheel is not the geometrical equation given above. Express the difference in the geometries of the rotating and non-rotating wheel, i.e. $C - C_0$.

Course Concepts *Activity 14*

The word *geometry* literally means the measure of the Earth. As you know, according to plane geometry the sum of the angles of a triangle is 180º.

A. Assume the Earth is a perfect smooth sphere. Imagine you construct a triangle on the *surface* of the Earth. What is the sum of the angles of this triangle? You may wish to consider an isosceles triangle with the apex at the North Pole and the base along the equator.
B. From Part A, you may have discovered that geometry on a spherical surface is different than the geometry on a plane. To describe the geometry around a massive object, which geometry is the best bet to be correct? Explain.

This Student Guide was written by David M. Harrison, Dept. of Physics, Univ. of Toronto in January 2009.

Activity 3 is based on Rachel E. Scherr, Peter S. Shaffer, and Stamatis Vokos, "The challenge of changing deeply held student beliefs about the relativity of simultaneity," American Journal of Physics **70** (12), December 2002, 1238 – 1248.

I learned about the geometric approach to Special Relativity of Activity 8 from Edwin F. Taylor and John Archibald Wheeler, **Spacetime Physics** (W.H. Freeman, 1963). This classic is highly recommended.

Activities 1, 7 and 9 also appear in David Harrison and William Ellis, **Student Activity Workbook** that accompanies Hans C. Ohanian and John T. Markert, **Physics for Engineers and Scientists**, 3rd ed. (W.W. Norton, 2007).

Activities 11 and 12 also appear in Mechanics Module 3.

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