Mechanics Module 5
Student Guide

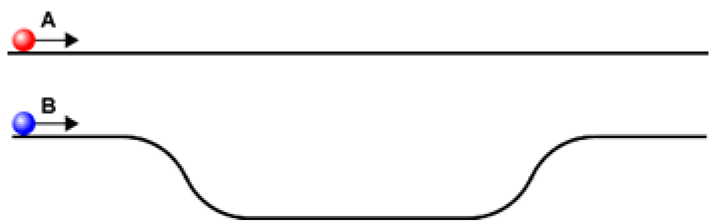
Concepts of this Module

- Kinetic, potential, elastic, and total energy
- Work

The Activities

Activity 1

Two balls are launched with equal initial speeds along tracks as shown. Friction and air resistance are negligible.



A. Predict which ball reaches the end of its track first.
B. You may check your prediction with a Flash animation at:

http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/ClassMechanics/RacingBalls/RacingBalls.html

A similar situation for skiers instead of balls is at:

http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/ClassMechanics/RacingSkiers/RacingSkiers.html

Of course, with animations one may program the wrong answer. A video of a real apparatus is at:

http://www.physics.umd.edu/lecdem/services/demos/demosc2/c2-11.mpg

Was your prediction correct?

C. Qualitatively explain the results of the race using conservation of energy
D. Qualitatively explain the results of the race using forces and accelerations.

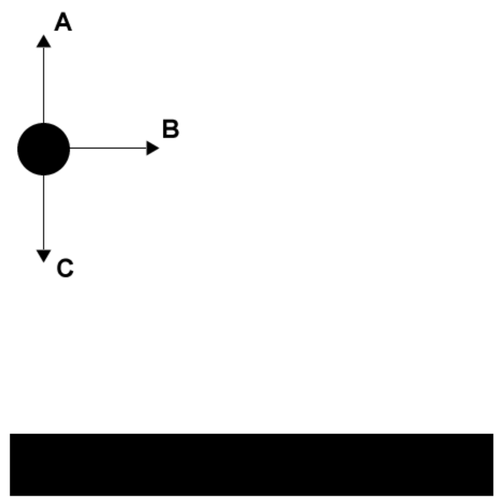
Course Concepts Activity 2

In Mechanics Module 3 Activity 11 you may have whirled a ball on a string in a vertical circle and noted that the speed of the ball at the bottom of the circle is greater than the speed at the top. At that time we did not explore why this is so very carefully.

1. Qualitatively explain the difference in the speeds using conservation of energy.
2. Qualitatively explain the difference in the speeds using forces and acceleration.

Course Concepts Activity 3

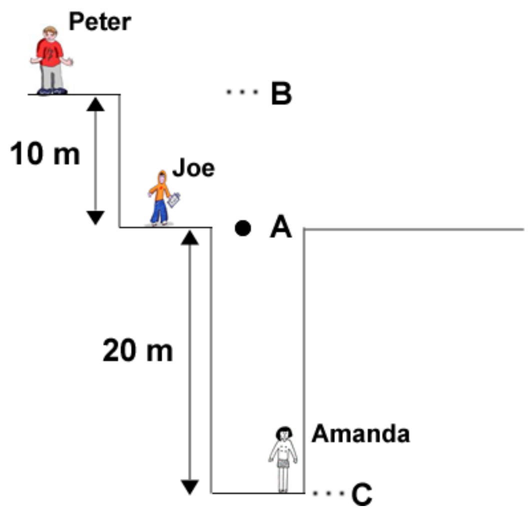
Three balls are the same height h above the ground and are fired with the *same* initial speeds v_0. Ball A is fired straight up, ball B is fired horizontally, and ball C is fired straight down. Air resistance is negligible.



A. Rank the speeds, from the largest to the smallest, of the three balls when they hit the ground. Explain.
B. Rank the time, from the largest to the smallest, it takes the three balls to hit the ground. Explain.

Course Concepts Activity 4

Joe is standing on the ground, Peter is standing on a 10 m high cliff, and Amanda is at the bottom of a 20 m deep pit, as shown. All three are using coordinate systems with the vertical axis directed up.



Joe's coordinate system has the zero of the vertical axis at ground level.

Peter's coordinate system has the zero of the vertical axis at the height of the cliff.

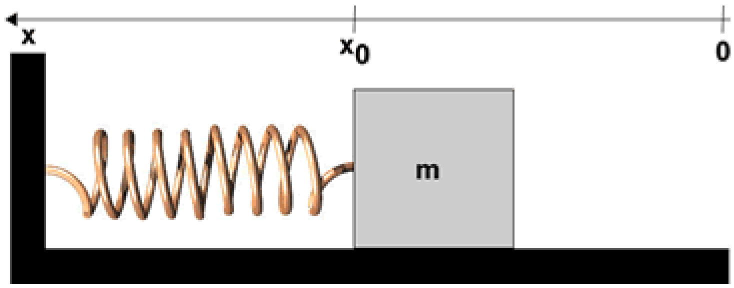
Amanda's coordinate system has the zero of the vertical axis at the bottom of the pit.

A ball of mass *m* is initially at rest at ground level, Position A above the pit.

A. What is the gravitational potential energy of the ball for Joe, for Peter, and for Amanda?
B. The ball is then raised to the height of the cliff, Position B, and is held at rest. What is the gravitational potential energy of the ball at Position B for Joe, for Peter, and for Amanda?
C. The ball is then released from rest and strikes the ground at the bottom of the pit, Position C. What is the gravitational potential energy of the ball at Position C for Joe, for Peter, and for Amanda?
D. What is the speed of the ball at Position C for Joe, for Peter, and for Amanda?

The following is used in Activities 5 - 8

A horizontal spring has an equilibrium position x_0. When the mass *m* is at position x_0 as shown the spring exerts no force on it. When the spring is either stretched or compressed, the position of the mass is *x* and the force the spring exerts on the mass is:



$$F = -k(x - x_0) \qquad 4.1$$

We assume an ideal spring and negligible air resistance. If the mass is oscillating, the mechanical energy is conserved and equal to:

$$\frac{1}{2}mv^2 + \frac{1}{2}k(x - x_0)^2 \qquad 4.2$$

Course Concepts Activity 5

In the above figure we have chosen a coordinate system that points from the right to the left.

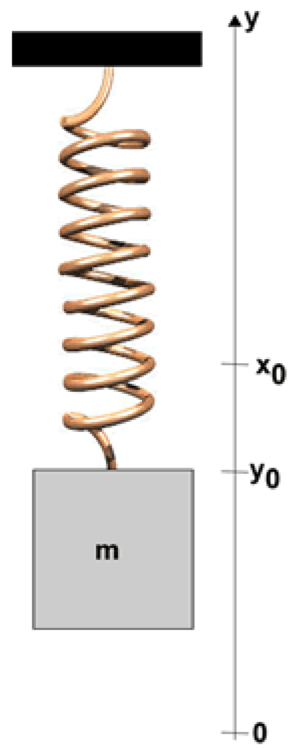
A. For values of $x > x_0$ does the force point in the $+x$ or the $-x$ direction?
B. For values of $0 < x < x_0$ does the force point in the $+x$ or the $-x$ direction?
C. For values of $x < 0$ does the force point in the $+x$ or the $-x$ direction?

Activity 6

The term $\frac{1}{2}k(x - x_0)^2$ is the *elastic potential energy* of the spring. Explain in your own words where the $\frac{1}{2}$ term comes from. There are at least two ways that you may wish to think about this.

1. In a graph of the force versus the distance, what physical quantity is given by the area under the graph?
2. What is $\int F\, dx$?

Activity 7



Now the same spring that is discussed in the introduction to Activities 5 – 8 above is suspended vertically. You may wish to note that the coordinate axis is now labeled *y*, while in the introduction above it was labeled *x*. The position labeled x_0 to the right is the same equilibrium position of the spring as before.

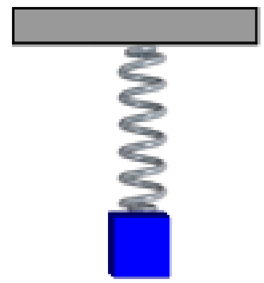
A. If the mass is at position x_0, the equilibrium position of the spring, draw the free body diagram of the forces acting on the mass.
B. At this position what is the net force acting on the mass?
C. If the mass is at some position y_0, the net force on the mass is zero. Draw the free body diagram of the forces acting on the mass.
D. What is the expression for y_0 in terms of m, g, k, and x_0?
E. What is the total vertical force acting on the mass as a function of k, y, and y_0?
F. What is the total mechanical energy when the mass is oscillating?

Activity 8

If you have not already done so, you should look at the introduction to Activities 5 – 8 that is above.

Suspend the supplied mass from the supplied spring. Gently lift the mass a small amount and release it, so that it bobs up and down. The oscillations should be small enough so that the motion is smooth and the spring is always vertical and stretched. Place the Motion Sensor under the mass with the transducer pointing up so it tracks the position of the mass. Later you will learn how to describe the position as a function of time. Here we will begin to explore oscillatory motion and look at the total mechanical energy of the system.

Collect distance-time data for the mass when it is vertically oscillating. Be aware that the Motion Sensor has a switch which adjusts the width of the beam; one of the switch settings will probably result in better data than the other. Recall that the Motion Sensor can only measure distances greater than 0.15 m, so you should be sure to make the amplitude of the oscillations small enough that the mass never gets closer to the Motion Sensor than 15 cm.



- Set the vertical position of the mass-spring position so that when the mass is oscillating the minimum distance from the Motion Sensor is **close** to but greater than 0.15 m. Try to have the mass moving only up and down.
- Set the Motion Sensor for the wide beam. On some units this is indicated by an icon of a person. You may wish to try the other beam setting.
- After starting the *MotionSensor.vi* software, set the sample rate to about 60 samples per second. You may wish to try a different sampling rate to see if it improves the quality of your data.
- Collect data for just a few oscillations.

Here are some tips for analyzing your data:

- It is likely that there will be noise in your values of the distance. These propagate to even greater noise in the displacement, velocity and acceleration. Use the cursors in the main Distance-Sample plot to select a reasonably clean set of data encompassing at least a bit more than half of one complete oscillation. The velocity-time graph is often particularly useful in determining the "region of interest" that you wish to keep. Sometimes the data will be so noisy that it is a good idea to take another set.
- The acceleration plot will be particularly noisy. By default this plot displays all of the values. You can adjust the minimum and maximum values of the plot to show the main features of the data without showing any noisy values by double-clicking on the minimum or maximum value of the vertical axis, putting in a new value, and pressing Return on the keyboard.

You will also want to be aware that the mass oscillates about some position y_0 but because the mass is hanging from the spring, this position is not the equilibrium position of the spring x_0. The difference between y_0 and x_0 is explored in Activity 7, which is not required for this Activity.

Now the Activities:

A. From your data, what is the value of y_0?
B. When the mass is at y_0, is its speed a maximum or a minimum?
C. What is the value of the speed when the mass is at y_0? Try to account for the noise in the plot by assigning an error to the value.
D. When the mass is at y_0, is its acceleration a maximum or a minimum?
E. When the mass is at y_0 what is the value of its acceleration? Try to account for the noise by assigning an error to the value.
F. From your data what is the maximum amplitude of the oscillation? When the mass is at this position, is its speed a maximum or a minimum?
G. What is the value of the speed when the mass is at its maximum amplitude? What is the error in this value?
H. When the mass is at its maximum amplitude is its acceleration a maximum or a minimum?
I. What is the value and error of the period of the oscillation? What is the value for the spring constant k and the error in this value?
J. When the mass is at y_0 what is the total mechanical energy? When the mass is it the maximum amplitude of the oscillation what is the total mechanical energy? From your data is mechanical energy conserved within errors? Explain.

Course Concepts ***Activity 9***

For an ideal spring-mass system we may write Newton's 2nd Law as:

$$F = -kx = ma \tag{9.1}$$

If one knows *integral calculus* then Eqn. 9.1 can be integrated to show that the mechanical energy E_{mech} is conserved, where:

$$E_{mech} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \tag{9.2}$$

In fact, integral calculus was invented by Newton (and independently by Leibniz) to do just this sort of Physics calculation.

Here you will use only dimensional and unit analysis, Newton's 2nd law, and the calculus of *derivatives* to show the relation between these equations.

Let us assume that the elastic potential energy U_S somehow depends on the spring constant *k* and how much the string is stretched from its equilibrium position *x*. Then is has a form:

$$U_S = d\,k^e\,x^f \tag{9.3}$$

where *d*, *e*, and *f* are dimensionless numbers. Then the mechanical energy is:

$$E_{mech} = \frac{1}{2}mv^2 + d\,k^e\,x^f \tag{9.4}$$

You will determine the values of *d*, *e*, and *f*.

A. The unit of U_S must be *energy*, and the unit of the spring constant *k* is *force/distance*. What must be the values of *e* and *f* in Equation 9.4? Rewrite Equation 9.4 with these values of *e* and *f*.
B. If the mechanical energy is conserved what must be the value of $\frac{dE_{mech}}{dt}$? Explain.
C. Calculate $\frac{dE_{mech}}{dt}$ from the equation of Part A, and set the result equal to your answer to Part B. Compare to Eqn. 9.1 to determine the value of *d*. Is your result consistent with Eqn. 9.2?

Course Concepts *Activity 10*

In Mechanics Module 2 Activity 9 you determined the angle with the horizontal, θ, that the track must make if the cart is to roll down it with constant speed. You will need to use your data from Module 2 Activity 9 for this Activity.

As the cart rolls down the track is mechanical energy conserved?

If your answer is *Yes*, explain.

If your answer is *No*, where did the energy go? How much mechanical energy is lost if the cart travels 1 m down the track?

Course Concepts *Activity 11*

Work is a word that is used both in Physics and in everyday life. Although the meanings of the word in these two contexts are similar, they are not identical. In 5 minutes or less think of as many uses as you can of the word *work* in your everyday life that do not correspond to the Physics definition, and illustrate each by using it in a complete sentence.

Course Concepts *Activity 12*

The relation between the *s* component of a force acting on an object, F_s, and the potential energy U is:

$$F_s = -\frac{dU}{ds} \qquad (11.1)$$

Note the minus sign.

On the next page are some other equations from the textbook with minus signs:

$$x - x_0 = \frac{1}{2}(v_0 + v)\,t$$

$$y = v_{0y}t - \frac{1}{2}gt^2$$

$$\mathbf{F}_{1\,on\,2} = -\mathbf{F}_{2\,on\,1}$$

$$F = -kx$$

$$W = -\Delta U$$

Are any of these minus signs conceptually the same as the one that appears in Eqn. 11.1? Explain.

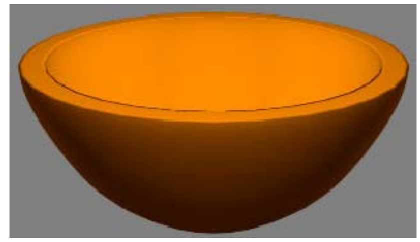
Course Concepts *Activity 13*

As you may already know Leibniz, a contemporary and bitter rival of Newton, believed that the "vis viva" mv^2 was the most crucial concept for understanding mechanics. Newton believed that the momentum $m\vec{v}$ was the most important quantity.

A. Now, we usually use one-half of the vis viva, ½ mv^2, and call it the kinetic energy. In your own words describe how the factor of one-half arises in the definition of kinetic energy.
B. Who do you think was right about the most important quantity: Newton or Leibniz? Why?

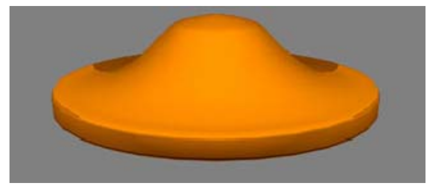
Course Concepts Activity 14

A popular toy is part of a hollow rubber sphere that pops when inverted and dropped. It is often called a “popper.” Cock the supplied popper a few times. You may find you need to curl the edges down a bit to get it to stay cocked.



Uncocked

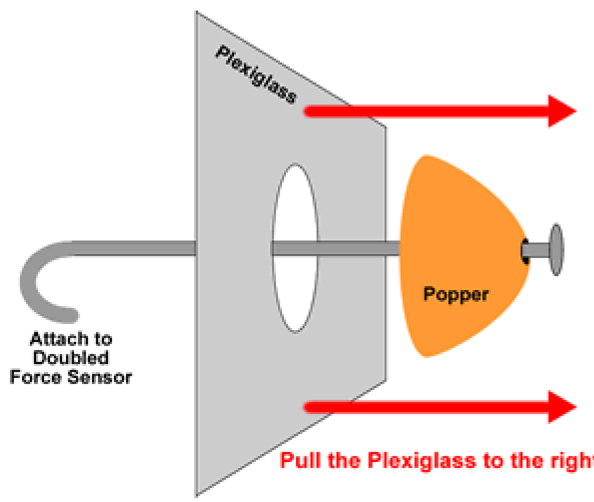
You will be using two Force Sensors in parallel to measure the force needed to cock the popper. The program that reads the two Force Sensors and displays their sum is named `ForceSensorsDoubled`. For more details see the Force Sensor manual at:



Cocked

http://www.upscale.utoronto.ca/Practicals/Manuals/Equipment/ForceSensor/ForceSensor.pdf

You will use the supplied “hook” and piece of Plexiglas with a hole in it as shown. Be sure to center the popper on the hole in the Plexiglas, or the popper can get pulled through the hole when it is cocked.

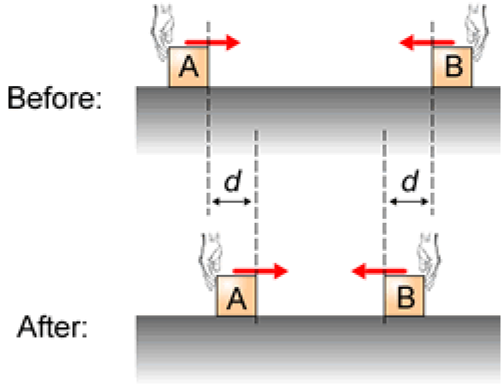


A. Use the doubled Force Sensors to estimate the force needed to cock the popper. Estimate the total work you need to do to cock it. Where did the energy go?

B. When placed on the floor some of the poppers uncock themselves almost immediately and fly straight up into the air. Others need to be dropped onto the floor from a height of a few centimeters. You do not want to launch it from the tabletop: it will often hit the ceiling. Determine whether your popper launches itself spontaneously or needs to be dropped. Repeat a few times, and note how high above the floor it was dropped from, if applicable, and how high it flies vertically up into the air.

C. Describe all of the energy transformations that occur as you cock the popper and then have it fly up into the air. Is the total energy conserved throughout all of these transformations? Do some rough calculations to justify your answer.

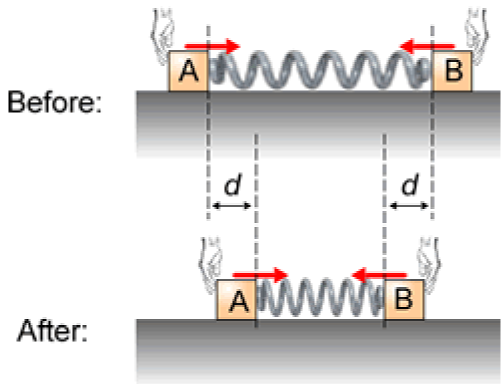
Activity 15



Two equal mass blocks are initially at rest and sitting on a frictionless surface. A hand exerts a force of magnitude *F* on block A, which pushes it to the right by a distance *d*. Another hand exerts of force of the same magnitude *F* on block B, which pushes it to the left the same distance *d*.

A. Is the sign of the work W_A the hand on the left does on block A positive, negative, or is it zero? Explain.
B. Is the sign of the work W_B the hand on the right does on block B positive, negative, or is it zero? Explain.
C. Consider the blocks A and B as the system. Is the sign of the net external work W_{ext} done by the hands on the system positive, negative, or is it zero? Explain.

Activity 16

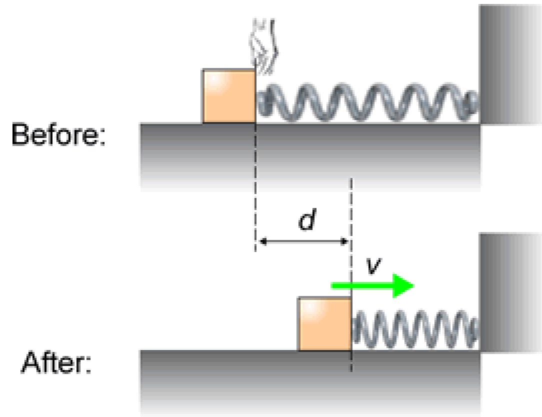


Two equal mass blocks are initially at rest and sitting on a frictionless surface. The blocks are connected by a massless ideal spring that is initially at its equilibrium position. A hand exerts a force of magnitude *F* on block A, which pushes it to the right by a distance *d*. Another hand exerts of force of the same magnitude *F* on block B, which pushes it to the left the same distance *d*. Consider the blocks A and B and the spring as the system.

A. Is the sign of the net external work W_{ext} done by the hands on the system positive, negative, or is it zero? Explain.
B. Is the change in the total energy ΔE_{tot} of the system positive, negative, or is it zero? Explain.

Activity 17

A block is initially at rest on a frictionless surface. It is connected to a massless ideal spring which is stretched a distance *d* from its equilibrium position. The other end of the spring is fixed to a wall. The block is kept in place by a hand. The hand releases the block and it moves to the right. In the **After** figure, the spring is at its equilibrium position and the block is moving to the right with speed *v*. Consider the block and spring as the system.



A. Is the sign of the work *W* done *by the wall* on the system positive, negative, or is it zero? Explain.
B. Is the change in the total energy ΔE_{tot} of the system positive, negative, or is it zero? Explain.

This Student Guide was written by David M. Harrison, Dept. of Physics, Univ. of Toronto, in the Fall of 2008. Revisions by David M. Harrison and Jason Harlow. Last revision: November 4, 2010.

Activity 3 is from Randall Knight, Student Workbook that accompanies the 1st edition of **Physics for Scientists and Engineers** (Pearson Addison-Wesley, 2004), Section 10.3, Activity 10. Activity 14 is similar to Priscilla Laws et al., **Workshop Physics Activity Guide** (John Wiley, 2004), Unit 11, 11.7. Activities 15 – 17 are from Beth A. Lindsey Paula R. L. Heron, and Peter S. Shaffer, American Journal of Physics **77**(11), November 2009, 999 – 1009.