Fluids Module
Student Guide

Concepts of this Module

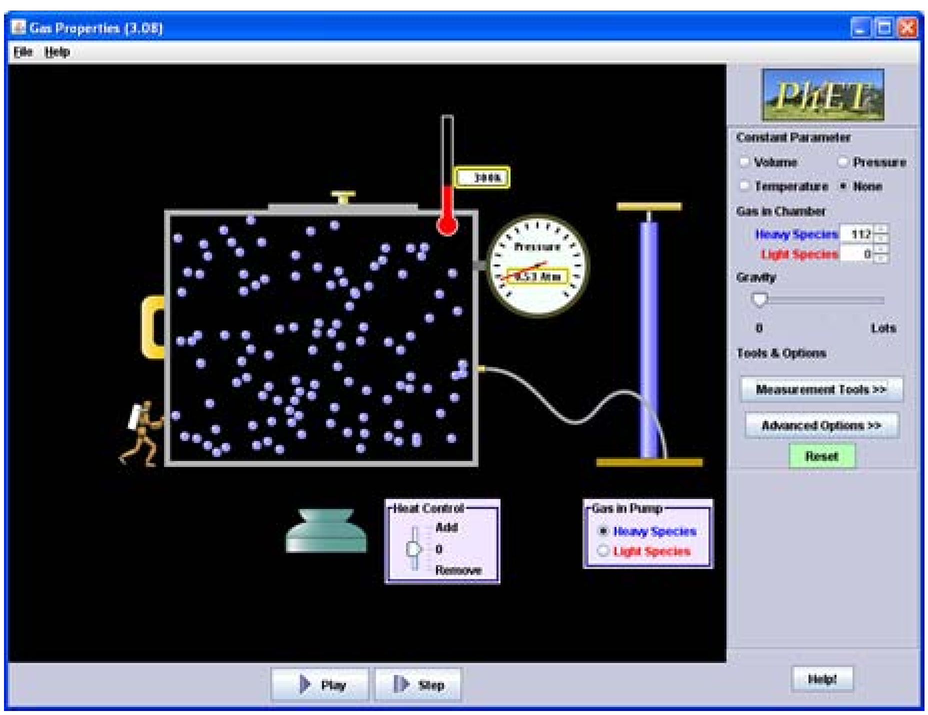
- Fluids
- Pressure
- Buoyancy
- Fluid dynamics

The Activities

Activity 1

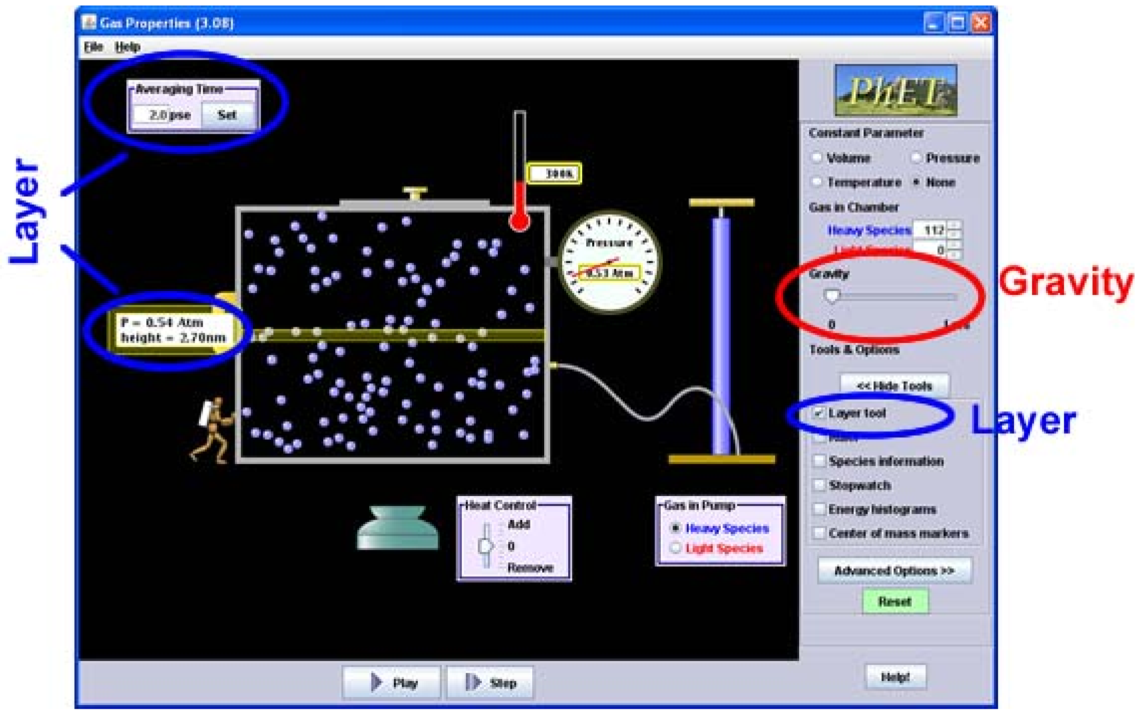
Open the `gas-properties.jar` animation which is located at `feynman:public/Modules/Fluids`. There are many useful ways to use this animation, and we will only draw you attention to a couple of things that you may wish to do; you are encouraged to explore further.

Here is a screen shot of the default animation after some *Heavy Species* molecules have been pumped into the container:



A. You will notice that the reading of the Pressure gauge is not constant. Explain why this is so. What would be necessary for the pressure reading to be more constant? How would you present a value for the pressure that also expresses your observed variations?

There are many options for controlling the animation. We shall describe two of them.



1. By default the acceleration due to gravity g is zero. You may introduce a non-zero value of g with the **Gravity** slider.
2. By clicking on the **Measurement Tools** button you may turn on the **Layer tool**. This tool measures the pressure in the gas at a specified height; you may drag the position of the measurement with the mouse. You can also specify the time over which the value of the pressure is averaged.

Here are some suggested explorations.

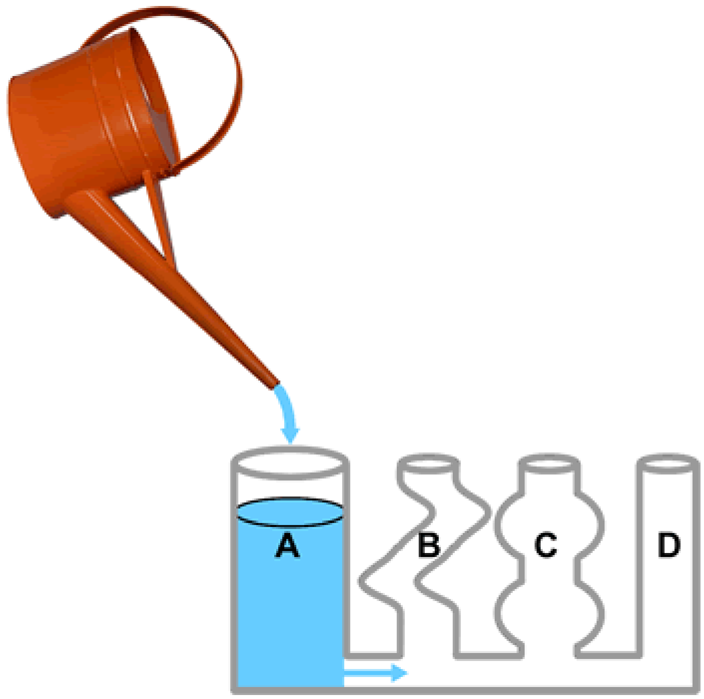
B. Use the **Layer tool** with various settings of the Averaging Time. Describe what happens. If this was not part of your answer to Part A, should it have been?

C. With **Gravity** set to 0, predict how the pressure in the gas varies with height. Check your prediction using the **Layer tool**. Were you correct? If the pressure varies with height, does it vary is the height, the height squared, one over the height, or what?

D. Introduce a non-zero **Gravity**. Predict how the pressure varies with the height. Check your prediction. Were you correct? If the pressure varies with height, does it vary is the height, the height squared, one over the height, or what?

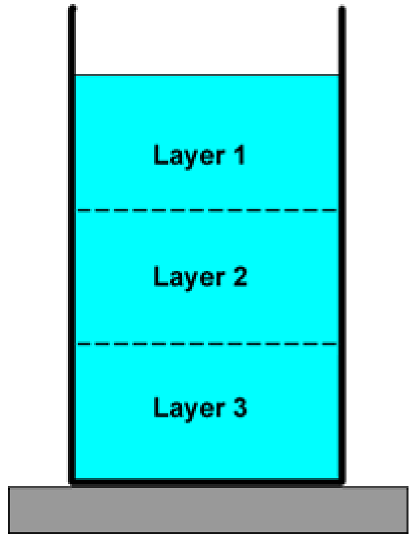
Course Concepts *Activity 2*

Cylinder **A** is being filled to the level shown. As the water is added to the cylinder it flows along the horizontal pipe and up **B**, **C** and **D**, which are all open at their tops.



Rank the heights of the water in **A**, **B**, **C**, and **D** when **A** is filled. Check your prediction using the supplied apparatus. Was your prediction correct? If yes, what physical principles did you use to make a correct prediction? If no, explain the actual result.

Activity 3



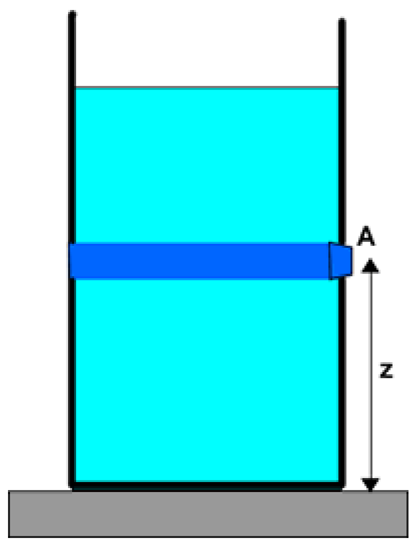
A rigid rectangular container filled with water is at rest on a table as shown. Two imaginary boundaries divide the water into three layers of equal volume. No material barrier separates the layers.

A. Draw a free body diagram for each layer. The label for each force should include:
   - A description of the force, and
   - The object on which the force is exerted, and
   - The object exerting the force.

B. Rank the magnitude all the *vertical* forces you have drawn for Part A, from the smallest to the largest. Explain how you determined the ranking.

C. Rank the magnitude of all the *horizontal* forces you have drawn for Part A, from the smallest to the largest. Explain how you determined the ranking.

Activity 4

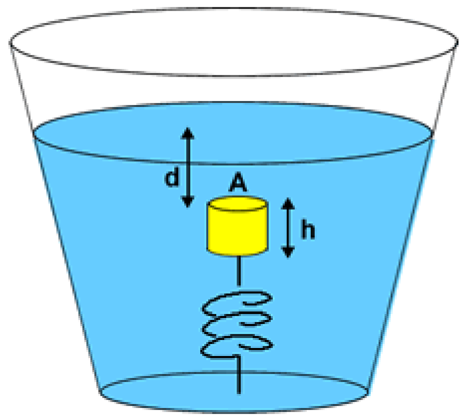


A small square hole of area **A** is cut in the side of the container of Activity 2. The centre of the hole is a height **z** above the tabletop. Consider the rectangular section of water of area **A** aligned with the hole, as shown.

A. Draw a free body diagram of all the forces acting on the rectangular section of water.

B. What will happen to the water just inside the hole?

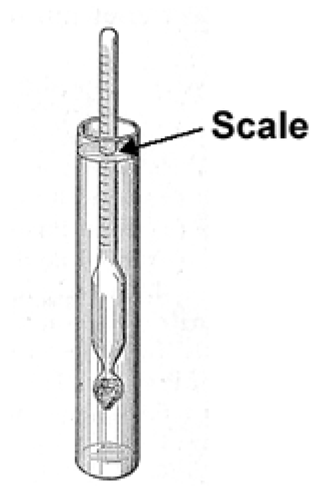
Activity 5



A bucket of water has a spring soldered to the bottom. Attached to the other end of the spring is a cylindrical cork of mass *m*, height **h** and area **A** which is stationary below the surface of the water, as shown. The top of the cork is a depth **d** below the surface of the water. The spring has a spring constant *k* and is stretched a distance **x** from its equilibrium position. The density of the water is ρ.

A. Draw a free body diagram of all the vertical forces acting on the cork. Evaluate the magnitude of those forces. Determine **x**, the amount that the string is stretched from its equilibrium position.
B. Imagine you are holding the bucket by its handle, which is not shown. You go to the top of the CN tower and step off, still holding the bucket. As you and the bucket fall towards the ground what is the motion of the cork? Does it move towards the bottom of the bucket, towards the top, or stay where it is? Explain.

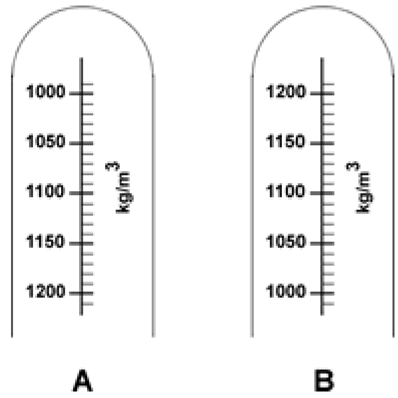
Activity 6



A *hydrometer* measures the density of a liquid. They are widely used to measure the alcohol content in the brewing of beer, the electrolyte content of battery acid, and more.

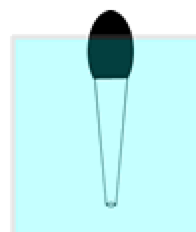
The device is placed in the liquid whose density is to be measured, and the density is read by the place on the scale where the surface of the liquid touches the stem.

On the next page is a close-up figure of two possible ways that the markings on the scale of the hydrometer can be arranged. Which of these arrangements are correct? Explain.



Course Concepts *Activity 7*

Please do this Activity with all the apparatus in the supplied 50×30×10 cm dishpan to minimize the water spilled onto the tabletop.



You are supplied with a beaker. You should fill it with water nearly to the top. Place the supplied medicine dropper in the water with the squeeze bulb on top. Suck enough water up into the medicine dropper that it *just barely* floats.

You are supplied with an empty 2 liter plastic pop bottle. Fill it to the brim with water. Transfer the filled medicine dropper to the water in the pop bottle.

Screw the top tightly on the bottle. Squeeze the bottle. What happens to the medicine dropper? What happens when you quit squeezing the bottle? Explain why squeezing the bottle and increasing the pressure of all the fluids within would cause the observed motion. This is called a *Cartesian diver*.

The supplied toothpicks make it easy to “fish” the medicine dropper out of the bottle.

When you are finished with this Activity, *carefully* empty all the water into the sink.

Course Concepts *Activity 8*

You may have noticed that the bubbles in a glass of a carbonated beverage (soda, beer, champagne, etc) accelerate as they rise from the bottom. Explain.

Activity 9

A ship is in a canal lock, which is only a little bit larger than the ship itself. The ship is loaded with steel ingots, which are large bars of steel. The crew becomes angry with the captain of the ship and throws the steel ingots overboard into the water of the lock.

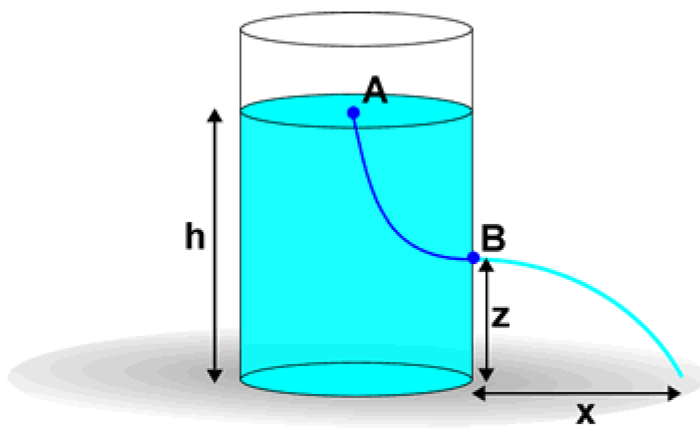
Does the level of the water in the lock rise, lower, or stay the same?

Check your prediction. You are supplied with a plastic tank which is about 30×20 cm and 12 cm deep, and a dishpan which is about 50×30 cm and 10 cm deep. Place the plastic tank in the dishpan and fill the tank about half-way with water. Place the supplied weight in the bottom of the supplied plastic boat and gently place it in the water. You may mark the height of the water in the tank with a small piece of masking tape. Carefully lift the boat out of the water, place the weight at the bottom of the tank, and put the boat back in the water.

When you are finished with this Activity, *carefully* empty the water into the sink.

Activity 10

A cylinder with water of height **h** has a small hole cut in the side at height **z**. The water strikes the ground at **x**. The figure shows the streamline from the top of the water at **A** to just outside the hole **B**. Bernoulli's equation tells us that the quantity $p + \frac{1}{2}\rho v^2 + \rho g y$ is conserved along the streamline, and so has the same value at points **A** and **B.**



If the hole is small, it is reasonable to approximate that the speed of the water at **A** is zero. Since point A and B are in contact with the outside air, it is reasonable to approximate that the pressure is the same at point A and B, that of atmospheric pressure in the room.

A. What will be the shape of the stream of water emerging from the hole until it strikes the ground?

B. Without using any equations, describe how the speed of the water at **B** varies with **z**, How will the distance **x** depend on **z**?

C. Use Bernoulli's equation and your knowledge of projectile motion to derive the answers to Part B. For what value of **z** will **x** be a maximum? What approximations are you making? Are those approximations reasonable?

D. You are supplied a cylinder with small holes cut in it at values of **z** = 0.75 **h**, 0.50 **h**, and 0.25 **h**, where the height of the water **h** is indicated by a mark on the cylinder. Place the cylinder in the supplied 50×30×10 cm dishpan: place it on one end of the dishpan with the holes pointing towards the other end of the dishpan. Fill the cylinder with water to the mark. As the water level drops appreciably add water. Is what you see consistent with your results from Parts B and C? Placing a strip of masking tape on the edge of the dishpan on which you can mark where the water lands is a convenient way to do this Part of the Activity.

When you are finished with this Activity, *carefully* empty the water into the sink.

Activity 11

You are given a length of flexible plastic hose. First take a small piece of paper or two, each about 5 mm square, and place them on the tabletop. This Activity will require two of you. One person holds the hose by one end, and the other person holds the hose about 50 cm from the other end. **Being careful not to hit anybody or anything**, the person holding the tube 50 cm from its end should whirl the free end of the hose in a *vertical* circle. A horizontal one will also work but makes it more difficult to avoid hitting somebody or something with the whirling tube. The person holding the end of the tube should hold it over the piece of paper on the tabletop.

The person whirling the hose should not grip it so tightly that it squishes.

What happened? Explain.

Please make our custodial staff happy, and pick up the bits of paper and put them in the blue box when you have finished this Activity.

Activity 12

When an object falls through a fluid, either a liquid or a gas, there are three forces that act on it:

1. The downward force due to gravity, $\vec{F}_G$. This is the *weight* of the object.
2. An upward buoyant force, $\vec{F}_B$. As Archimedes realized over 2,000 years ago, this is equal to the weight of the fluid that the object displaces.
3. An upward drag force, $\vec{F}_D$.

In this Activity we will concentrate on the drag force exerted on a sphere falling through a fluid. We assume that the surface of the sphere is perfectly smooth. We will use the following variables in the discussion:

- *r*: the radius of the ball.
- *v*: the instantaneous speed of the ball.
- ρ: the density of the fluid.
- η: the dynamic viscosity of the fluid. This is sometimes called *liquid friction*. It is measured in units of pressure × time.

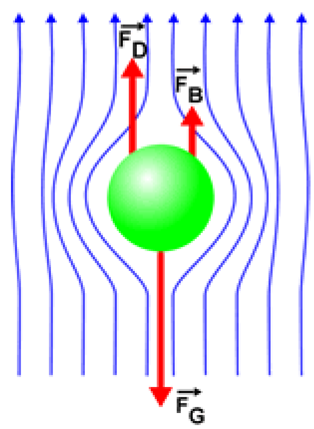
Here are some values for the density and viscosity of various fluids.

Fluid	Density (kg/m^3)	Viscosity (Pa-s)
Superfluid	--	0
Air (20 ºC)	1.2	1.82×10^{-5}
Water (20 ºC)	998	1.00×10^{-3}
Olive Oil (88 ºC)	914	4.32×10^{-2}
Glycerine (20 ºC)	1260	6.58×10^{-1}
Honey (20 ºC)	1,500	5.00

There are various ways that the fluid can flow around the sphere. If the speed of the ball is small, the flow is "smooth" or "laminar". In this case it turns out that the drag force is proportional to the speed.

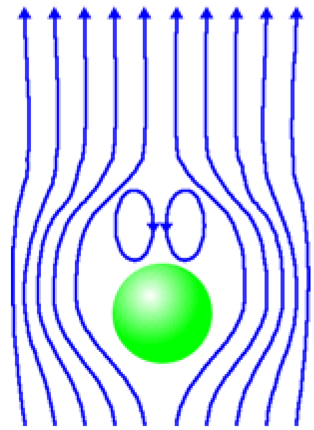
$$F_D \propto v$$

This was first shown by Stokes in 1851. This will be explored further in Part E.



When the ball is going faster, vortices develop in the fluid behind the ball as shown. In this case the drag force is approximately proportional to the speed squared.

$$F_D \propto v^2$$



Note that in both of these cases, as the ball's speed increases the drag force increases. Thus at some point the drag plus buoyant forces approaches the magnitude to the weight of the ball, so asymptotically there is zero net force acting. Then the speed of the ball becomes constant: the value of the speed is called the *terminal velocity*. For a sky diver falling face down with arms and legs outstretched, the terminal velocity is about 55 m/s. If the sky diver falls feet first, feet together and arms close to the body, the terminal velocity goes up to about 90 m/s. When the sky diver opens the parachute, the drag force goes way up and the terminal velocity falls to about 5 m/s.

Whether the fluid flow around the ball is laminar or turbulent turns out to depend only on a single dimensionless number, the *Reynolds number Re*.

$$\mathrm{Re} \equiv \frac{r\rho}{\eta} v$$

Re	Type of Flow
≤ 1	Laminar
$1 - 1 \times 10^3$	Transition
$1 \times 10^3 - 1.5 \times 10^5$	Turbulent

Note that for constant r, ρ, and η the Reynolds number is proportional to speed. Therefore when a ball is dropped from a large height, Re increases until the terminal velocity is reached.

When the Reynolds number reaches $\sim 1.5 \times 10^5$ the forces on the fluid near the ball become extreme, and both the wake and the layer of fluid right next to the ball become turbulent. This causes a sudden change in the way the luid flows around the ball, and the turbulent wake becomes narrower. When this happens, the drag force drops and the acceleration of the ball increases. This is called the *drag crisis*. As the speed increases further, the drag force resumes increasing with speed.

A. A ball of radius r is falling through a fluid and at some time has an instantaneous speed v. A second ball of radius $2r$ is falling through the same fluid. At what instantaneous speed will the second ball have the same flow pattern of fluid around it as the first ball?

B. Here is the URL of a Flash animation of dropping a ball from the CN Tower:

http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/FluidDynamics/BallCNTower/BallCNTower.html

The above link is to a fixed size animation which works nicely if only one person it viewing it. For use in the Practical itself a version which can be resized to be larger so that the entire Team can see it is better. Here is a link to such a version:

http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/FluidDynamics/BallCNTower/BallCNTower.swf

Open the animation and explore how it works.

For the keen some details about this animation and the one you will explore in Part D are in the Appendix.

C. For air, in SI units the Reynolds number is:

$$\mathrm{Re} \equiv \frac{r\,\rho}{\eta} v \cong 70{,}000\, r\, v$$

For the billiard ball, 5-pin bowling ball, and 10-pin bowling ball calculate the speed for which the drag crisis occurs. Are these results consistent with what you see in the animation of Part B?

D. Here is the URL of a Flash animation of dropping a ball in a liquid:

http://www.upscale.utoronto.ca/GeneralInterest/Harrison/Flash/FluidDynamics/ViscousMotion/ViscousMotion.html

As with the animation of Part B, you may access a resizable version at:

http://www.upscale.utoronto.ca/GeneralInterest/Harrison/Flash/FluidDynamics/ViscousMotion/ViscousMotion.swf

Open the animation and explore how it works.

E. For small Reynolds numbers, so the fluid flow is laminar, the drag force is:

$$F_D = 6\pi\,\eta\, r\, v$$

You may be surprised by the fact that the density of the fluid does not appear in this equation. When the sphere is at terminal velocity the net force is zero:

$$F_D = F_G - F_B$$

Therefore:

$$v_{terminal} = \frac{F_G - F_B}{6\pi \eta r}$$

For the animation of Part D, set the following values:

- r = 20 mm
- η = 5850 mPa-s
- ρ_{liquid} = 1500 kg/m^3
- ρ_{ball} = 5000 kg/m^3

Note the values of the ball weight and buoyant force and their difference. Run the animation and note the terminal velocity.

Now set the radius of the ball to 25 mm. Adjust the viscosity of the liquid so the drag force will be the same as the previous case. Adjust the liquid and ball densities so the ball weight minus the buoyant force is about the same as the previous case; you are unlikely to find values of the densities which are *exactly* the same, but can find ones that make the value almost the same. Does the animation match the theory? In particular is the motion of the ball the same for these two cases?

Appendix

Although the details of how the animations of Activity 12 work are not important for your learning of fluid dynamics, here we "lift the hood" to discuss the internals of the animations.

Except in the limit of laminar flow, the theory of drag forces is not easily solvable. Thus the animation uses a mixture of experimental data and some heuristic formulae that describe the data reasonably well. It turns out to be convenient to describe the drag force in terms of a *drag coefficient* C_D(Re), which is a function only of the Reynolds number. Then the drag force is:

$$F_D = C_D(\mathrm{Re})\, \rho_{liquid}\, r^2 v^2$$

For laminar flow (Re $\leq$ 1):

$$C_D(\mathrm{Re}) = \frac{6\pi}{\mathrm{Re}}$$

For larger values of the Reynolds number, experimental data on the dependence of the drag coefficient on the Reynolds number must be used. The data used in the animation is adapted from H. Edward Donley, UMAP Journal **12**(1), 47 (1991), http://www.ma.iup.edu/projects/CalcDEMma/drag/drag7.html.

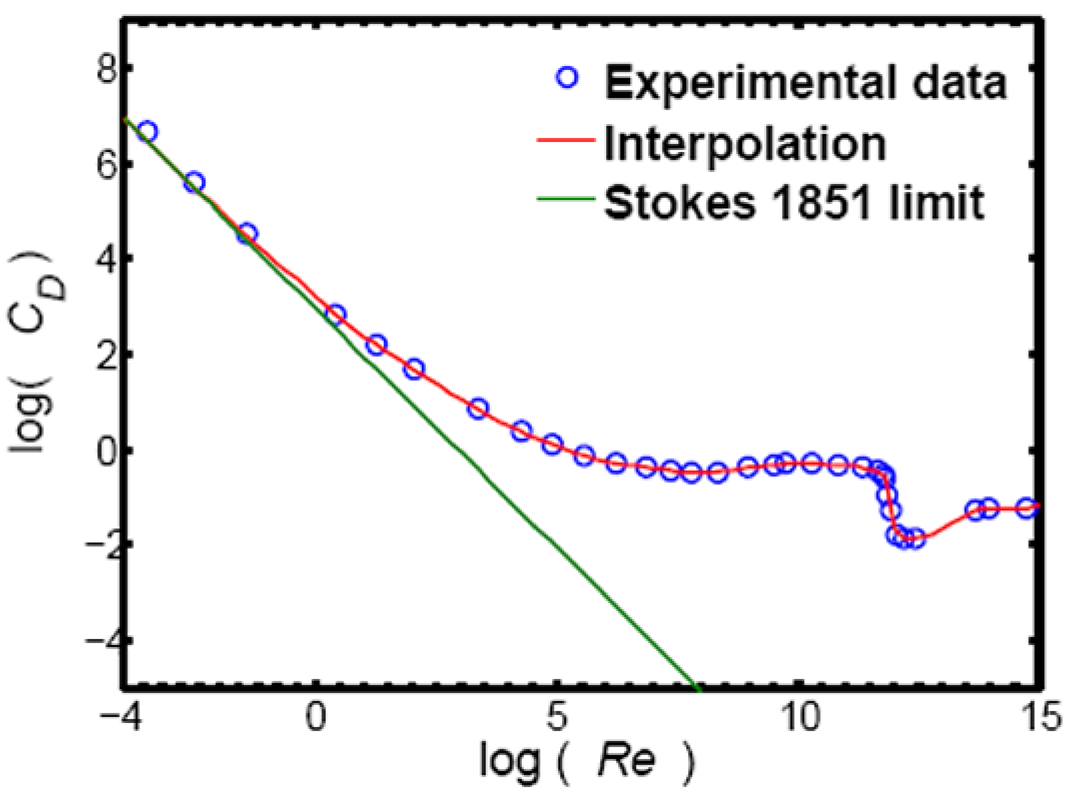
To parameterize this data involves some truly ugly equations. We used forms by John Versey and Nigel Goldenfleld, "Simple viscous flows: From boundary layers to the renormalization group", Rev. Mod. Phys. **39**(3), 883 (2007), http://rmp.aps.org/browse.

The Donley data and the Versey and Goldenfleld interpolation of it is shown on the next page.

The turbulent flow case ($1 \times 10^3 < \mathrm{Re} < 1.5 \times 10^5$) corresponds to the part of the above plot where the drag coefficient is approximately constant independent of the Reynolds number. Note that, despite the notation used in the axes of the figure, the values are the *natural* logarithms of the values.

The drag crisis is when the drag coefficient suddenly drops.

Once the drag force for a given speed has been determined, then we know the net force acting on the ball and hence its acceleration. We use a *numerical approximation* to find the motion of the ball. The method is similar to one you may have explored in the **Numerical Approximation** Module. For a time step *dt*.



1. From the acceleration, calculate the new speed of the ball:
   $v_{new} = v_{old} + a \times dt$
2. From the new speed calculate the new position of the ball:
   $y_{new} = y_{old} + v_{new} \times dt$
3. From the new value of the speed calculate the new drag force and then the new acceleration of the ball.
4. Go to 1 and repeat.

The above scheme turns out to not be accurate enough for our animation, so an extension of it called a *4th order Runge-Kutta* is used. It turns out that for this calculation to be stable we must iterate the Runge-Kutta 10 times for every frame of the animation. Since the animation runs at 12 frames per second, this means that the time step *dt* is 1/120 = 0.17 s.

This Student Guide was written by David M. Harrison, Dept. of Physics, Univ. of Toronto, May 2008.

The animation used in Activity 1 is from the Physics Education Technology (PhET) group at the University of Colorado, http://phet.colorado.edu/new/index.php. Activity 3 is based on Lillian McDermott et al., **Tutorials in Introductory Physics** (Prentice Hall, 20020, ST 219. Activities 6 and 9 are based on David M. Harrison and William Ellis, **Student Activity Workbook**, 3rd ed. (Norton, 2008), 18.4 and 18.6. The figure for Activity 6 is slightly modified from a figure from Wikipedia, http://en.wikipedia.org/wiki/Hydrometer, retrieved June 19, 2008. Activity 11 is from John Caranci of the Toronto District School Board.

Revised September 9, 2010, David M. Harrison and Lilian Leung.
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