In the 40-some years that I have been interacting with undergraduate Physics students, I have learned a little bit about how to effectively study by watching what works and what doesn’t work for students. In addition, in the past 20 years education research has taught me even more about how we learn. This document outlines some of the things that I have learned. Some of what appears below is applicable to studying subjects other than Physics, both in the sciences and in other disciplines, although I will concentrate on Physics.

Of course, the best way to do as well as possible on the tests and exam requires knowing the material on which you are being tested. Later in this document we will discuss some further ideas about how to do well on the tests and exam.

It turns out that a fair amount of data has been collected that destroys some common myths about effective studying that you may have heard. Below I will identify some of these incorrect ideas.

**General Principles**

Physics involves only a few powerful concepts. Once you fully understand those concepts, applying them to answer questions about a physical situation or solve a problem is easy. However, full understanding of those concepts takes some time. The data show that in Physics a last-minute cram is not nearly as effective as working at a steady pace each and every week. So the 1st general principle is:

1. Keep up with your studies

Some disciplines will require you to know a large number of facts for the tests and examinations. For those disciplines, a last-minute cram can actually work. It won’t work in Physics.

Physics is like building a house: we first build a foundation, and then build the 1st floor on the foundation, and then a 2nd floor on top of the 1st floor, etc. If the foundation is not strong, then the 1st floor may fall down. Similarly, if the 1st floor is poor the 2nd floor will be too. So the 2nd general principle is:
2. Be sure to understand the previous concepts before trying to learn the new ones.

A few years ago Harvard identified a large number of characteristics and behaviours of their 1st year students, and looked for correlations with academic success. The only thing that they found that was a good predictor of success was whether or not the student was a participant in a study group with his/her classmates. There is a large body of data that show that this is a general rule, not just for Harvard students, so the 3rd general principle is:

3. Join or form a study group.

A number of universities have “Learning Communities” to facilitate study groups and more. Further information about them is in the “To Learn More” section at the end of this document.

Some students learned a survival strategy in their high school Physics class of memorising all the equations. Then on the tests just find the right equation and “plug and chug” to the answer. This strategy will not work for University-level Physics. So:

4. Don’t try to memorise your way through the course.

A typical physicist’s education about, say, classical mechanics first consisted of learning about it in high school. Then their 1st year University Physics course devoted a lot of the term to studying classical mechanics again. In their 2nd or 3rd year of undergraduate studies they took a full half-course in classical mechanics. Then in their 1st year of graduate studies they took yet another half-course in classical mechanics. Each iteration of the course studied the topic in much greater depth and sophistication. Similarly, you may have studied many of the topics of this course in your high school Physics course. However the level in this course will be considerably deeper and more sophisticated than the high school version.

5. Don’t think that you already know the material of the course at an appropriate level.

A common model, used by both students and teachers, is that the process of learning is basically filling up the brain with knowledge. In this model, the learner need only passively absorb the information being presented by the teacher, textbook, lab or Practical, homework problem, etc. The data are clear that good learning doesn’t happen this way. To learn, you must expend time and effort. Often you will need to “wrestle” with the ideas. When you don’t understand, don’t just blow the concept off. So, our final general principle is:

The **Note Taking** section below discusses one way to make your learning more aggressive.

**Myth-busting 1**

Although keeping up with your studies, Principle #1 above, is important, it is common to see documents such as this one that recommend that you should study:

- In a quiet environment, with no music or other distractions. **WRONG**
- At the same time and place for every session. **WRONG**

The data are clear that neither of these recommendations are appropriate for most people. Studying in different environments at different times and in different ways is the most effective strategy.

These same studies also show something that was implied in recommending that you should study at different times. A single long study session is not nearly as effective as breaking up your studying into different sessions separated by a few days.

Another, perhaps related, reason why studying in a single long session is not a good idea is related to the biochemistry of the brain. The sort of thinking involved in studying requires that your brain’s metabolic rate is higher than when you are just resting. But the brain’s metabolism “burns” glucose, and after some period of time of high metabolic activity, the glucose in the brain becomes depleted. When this happens you cannot think at a high level! You can partially compensate for this by eating something with a good glycemic index, such as a protein bar.

**Note Taking**

**While Reading**

When we read a work of fiction, we typically read at about a page a minute. When reading a science textbook or course notes, this is much too fast to assimilate the large quantity of information that typically appears on each and every page. One good way to force ourselves to slow down and pay close attention is to take detailed notes while reading. The notes should be **complete**. In particular, it is worthwhile to:

- Write down all definitions.
- Write down a summary of all physical arguments leading to a conclusion.
- Write down all equations. When all of the steps between one equation and the next are not given, fill in those steps.
- Sketch all figures and diagrams. Include the labels and captions. Often the figures are more important than the words in understanding the topic being discussed.
Buying a ream (500 sheets) of cheap paper for your notes is usually a good investment.

What to do with the notes at the end is up to you. Some people just throw them in the recycling bin. Others find they form a useful study guide. What is important is the process of taking the notes while reading the course material. Part of that process is getting your hand, eye, and brain all coordinated. This coordination is a skill that will be very useful when you are writing the tests and exams for the course.

**During Class**

Often in class, we find ourselves “tuning out” the instructor. Taking notes in class helps you to focus on what is being said. These notes should also be as complete as possible.

Shortly after class, it is an excellent idea to copy the notes you took in class into a more complete form, adding anything that was said and done during the class that you didn’t have time to write down. The revised notes can often be a useful study guide later, so consider keeping them.

As with the notes you take while reading the textbook or course notes, the process of taking and revising the notes will also help in getting your hand, eye, and brain coordinated.

**Taking Notes in Class by Hand vs. Using a Laptop**

Since most of us can type faster than we can write, you may be thinking that taking notes with a laptop is a better idea than using a pencil (or pen) and paper: they will be more complete.

This has been looked at by Mueller & Oppenheimer in a study published in 2014 and since replicated. It turns out that, although notes taken on a laptop are more complete, those notes are less effective in terms of comprehension and retention than taking the notes by hand.

Later studies have compared writing on a tablet with a stylus to writing on paper with pen or pencil, and typing on a laptop. Typing notes on a laptop was the least effective.

Summarising:

- Taking notes in class using a laptop is not a good idea: write them out in longhand.

**Problem Solving**

One of the greatest physicists and teachers of the 20th century was Richard Feynman. He summarised the process of problem solving like this:
1. Write down the problem
2. Think very hard
3. Write down the answer

For most of the rest of us, a more complete list is necessary. Here is one such list:

1. **Form a model.** The real physical world is very complex, and almost always we need to form a simplified model of it to solve a particular problem. As George Fox wrote, “All models are wrong, but some are useful.” Explicitly writing down what model and simplifying assumptions are being made is often helpful.
2. **Visualise.** This is where most experienced problem-solvers spend most of their time. It is a crucial part of Feynman’s “Write down the problem” step. The visualisation can include free-body diagrams, graphs, sketches of molecules, pictorial representations, and more.
3. **Guess the answer.** Use any physical principle, intuition, symmetry, or conservation law that you can think of to guess the answer. A correct guess reinforces your instincts. A wrong guess brings the refreshment of surprise. Guessing the answer like this before going any further is called *Wheeler’s First Moral Principle* after John Archibald Wheeler, another great 20th century physicist and teacher.
4. **Solve.** Usually this step means casting the problem into one or more equations, and then solving the equations to get a final answer. Casting the problem into one or more equations is Physics; pushing the symbols around on a piece of paper with a pencil to get an answer is just mathematics.
5. **Assess.** Often we get a final answer and just stop, or perhaps check the answer against the answer in the back of the textbook or posted online without further thought. This is a poor idea. Instead, think about the answer first. Is it physically reasonable? Is it consistent with your guess? Are the units correct? Do the equations that you ended up solving make physical sense? Checking special cases, such as “turning off gravity” or making a mass zero or infinite, is extremely useful and may give you new insights into the physics.

It definitely takes some self-discipline to explicitly go through each of these five steps when solving a problem, but experience shows that the time spent is definitely worthwhile.

**Further Remarks About the Solve Step #4**

If a problem is numeric, often one solves some of the equations to get an intermediate one, and then uses the intermediate result to solve to get the final desired result. In such a case, you should **keep the problem algebraic until the very end**, when you put in the numbers. There are a number of reasons why this is important:

1. Putting in the numbers for the intermediate result, and then using that numerical value to get the final answer increases the chances that you will make an error in
punching all the numbers into your calculator. By waiting until the very end, you need only use your calculator once.

2. Often there will be cancellations or simplifications in the equations, so you end up with a much simpler algebraic expression at the end to evaluate.

3. By putting the final answer into algebraic form first, you can assess that equation to see if it makes sense.

An Example

You have some physical system and find the following equations that describe the situation.

\[ y_1 = \frac{1}{2} a_1 t^2 \]  \hspace{1cm} (1)

\[ v = a_1 t \]  \hspace{1cm} (2)

\[ a_2 y_2 = \frac{1}{2} v^2 \]  \hspace{1cm} (3)

You are to find \( y_1 + y_2 \) when \( a_1 = 39.2 \text{ m/s}^2 \), \( a_2 = 9.80 \text{ m/s}^2 \), \( t = 10.0 \text{ s} \). We will solve by putting in the numbers at each step, and then solve it again by putting in the numbers at the end.

Although Eqns. 1 – 3 suggest that this is a problem involving kinematics, that is not important for illustrating the process of getting to the answer.

Putting in the Numbers at Each Step

We can solve Eqn. 1 to get the value of \( y_1 \)

\[ y_1 = \frac{1}{2} a_1 t^2 = \frac{1}{2} (39.2 \text{ m/s}^2) \times (10.0 \text{ s})^2 = 1960 \text{ m} \]  \hspace{1cm} (4)

Next we can directly solve Eqn. 2 to get the value of \( v \)

\[ v = a_1 t = (39.2 \text{ m/s}^2) \times (10.0 \text{ s}) = 392 \text{ m/s} \]  \hspace{1cm} (5)

Then we can solve Eqn. 3 for \( y_2 \) using the value of \( v \) from Eqn. 5

\[ y_2 = \frac{v^2}{2 a_2} = \frac{(392 \text{ m/s})^2}{2 \times 9.80 \text{ m/s}^2} = 7840 \text{ m} \]  \hspace{1cm} (6)

So we can finally get the answer by combining Eqns. 4 and 6
\[ y_1 + y_2 = (1960 \text{ m}) + (7840 \text{ m}) = 9800 \text{ m} \]  

(7)

So this method of solving the problem requires doing four different calculations. There are lots of chances to make a mistake!

**Putting in the Numbers At the End**

From Eqns. 2 and 3

\[ y_2 = \frac{1}{2} v^2 = \frac{1}{2} \frac{a_2^2 t^2}{a_2} \]  

(8)

So

\[ y_1 + y_2 = \frac{1}{2} a_1 t^2 + \frac{1}{2} \frac{a_2^2 t^2}{a_2} = \frac{1}{2} a_1 t^2 \left( 1 + \frac{a_1}{a_2} \right) \]  

(9)

Now we can put in the numbers.

\[
y_1 + y_2 = \frac{1}{2} a_1 t^2 \left( 1 + \frac{a_1}{a_2} \right) = \frac{1}{2} (39.2 \text{ m/s}^2) \times (10.0 \text{ s})^2 \left( 1 + \frac{39.2 \text{ m/s}^2}{9.80 \text{ m/s}^2} \right)
\]

(10)

\[ = 9800 \text{ m} \]

The two methods give the same answer provided we didn’t make any mistakes. But by putting in the numbers at the end we got to the solution faster, and only had to use our calculator once. Further, using your calculator in a simple-minded way the first method required inputting 10 numbers, while the second method only required 6. So the chances of making a mistake are much much less by putting in the numbers at the end.

**Math & Physics**

Some students in introductory physics courses are worried about “all that math” in the course. If you are one of those students, you should be aware that in physics we use mathematics in two distinct ways:

1. As a language to describe the physical universe.
2. As a tool to solve problems.
We briefly referred to this in the Solve Step #4 of the problem solving strategy above.

Here is an example of math as a language to describe the world. The most famous equation in the world is probably Einstein’s $E = mc^2$. Here is a translation of that equation into English:

In Einstein’s theories of relativity the speed of light, $c$, is just a conversion factor for units. So, if we ignore units we can write the equation as: $E = m$. This says that “energy” $E$ and “mass” $m$ are just different names for the same thing, i.e. energy and mass are equivalent.

In the previous section when we discussed problem solving, we saw that one of the steps involves casting the physical situation into one or more equations: this is physics.

Then the equations are solved. This is *not* physics: this is just math. For example, when we did an example of solving some equations to get a final answer above, there are many software programs that can “do the math” as well or better than we can. And of course, the program doesn’t know anything about physics.

The final step in problem solving, assessing the answer, is again physics.

Most introductory physics courses for non-majors are not very interested in testing your ability to do mathematics, but are interested in you becoming proficient in using it as a language.

I will leave to the philosophers and linguists the question of whether the fact that physicists think in the language of mathematics influences the questions that they ask about the world and the answers that they come up with to those questions.

**Myth-busting 2**

Some students believe that doing a large quantity of problems, such as all the problems at the end of the chapters of the textbook, is a good way to do well in the course. There are data that show that this strategy is not effective. One study involved a Physics course where some students did over 2000 problems! It was found that after doing a reasonable number of problems, student performance didn’t increase by doing even more of them. So all those extra problems were a waste of time.

This same study found that the most influential factor in determining how the students did in the course was the problem solving and learning *strategies* that were used. Those strategies have been the primary focus of this document. In short:

- Studying *smart* is better than studying *lots*. 
Different Components of a Course

A typical science course consists of classes, textbook or course note readings, perhaps a practical, perhaps a tutorial, perhaps a laboratory, homework problems, and sometimes more. Each component of the course takes a somewhat different perspective on the same underlying content.

It is important to realise that each of these components of the course all have the same goal: to assist you in learning the content so that you may do well on the tests and exams. So, for example, when you are doing a laboratory or practical try to remember that what you are learning there is part of the process of understanding the material of the course. Similarly for all of the other parts of your course: they are all trying to help you understand the important concepts and facts of the course.

Do the Homework Yourself!

In introductory courses, regular homework assignments are assigned, collected, and count for a small part of your grade in the course. Whether the assignments are done on paper, or via a computer, it is easy to get the solutions from a “friend” and turn them in and get full credit. This is a terrible strategy.

In PHY131 at Toronto we use MasteringPhysics software to deliver homework problems. One feature of the software is that it tracks everything that you type and when you typed it. So we can identify students who enter the answer to a problem in less than a couple of minutes from when the problem was first shown to them. Perhaps some of these students had a printout of the question, and had solved it before logging in to MasteringPhysics. Perhaps some of them had solved the problem in a study group, and so one member of the group had taken a reasonable time from beginning to end, with the other members of the group able to enter the answer very quickly. However, it is reasonable to assume that the vast majority of these “fast” students had gotten the answer from somebody else and just copied it into the system.

When the overall performance of these “fast” students was looked at, it was found that their grades on the tests and examinations were almost a full letter grade less than the students who took a reasonable time to answer the questions. The conclusion is that doing the homework problems yourself really does help you learn the material, and that the learning will be demonstrated on the tests and examinations. This is true whether the assignments are on paper or via a computer. You can “do the math” and conclude that your final grade in the course will also be much higher if you solve the homework problems yourself!

In upper-year courses, we hope that students have figured this out, and typically homework assignments are given but not collected or marked.
Test and Exams

Answer the Question That is Being Asked

I often see students who on a test give a pretty good answer to a question that is not the one being asked. Of course, grades are only given for correct answers to the question that is being asked. Be sure to carefully read the question, so you are sure what the question is.

Setting Yourself Up to Succeed

When we are nervous, adrenaline and all sorts of other chemicals start rushing through our blood stream. If we let them, these chemicals will start thinking for us. But chemicals are stupid! One of your main goals during a test is to think with your very smart brain, not those stupid chemicals. Some ways to achieve this are:

1. Some time well before the test begins, stop studying! Instead, consider going for a walk. The last-minute cram will get all those chemicals going, and will be almost certain to cause more confusion than understanding. There are data that show that especially for conceptually based courses like Physics the last minute cram actually does more harm than good.
2. As you walk into the test, tell yourself: “I understand this material so well that this will be really easy.” Try to believe yourself! Of course, this is easier said than done, but nonetheless you want to try to achieve a state of calm confidence.
3. During the test, when you don’t think you know how to do a question don’t stress, just go on to another question. Later when you come back to the difficult question, you will often be pleasantly surprised to discover that you now know how to do it.
4. During the test, remember the advice you may have heard from your mother, grandmother, or maybe Aunt Tilly: good posture and good breathing.

You Snooze, You Win

A common phrase is “You snooze, you lose.” It turns out that in terms of learning this is not correct. Sleeping is important in allowing your brain to consolidate material that you have been studying. Therefore the common advice to get a good night’s sleep before a test or exam is an excellent idea, not only because you will then be rested and capable of your best work while taking the test, but also because sleeping will allow your brain to make connections and deepen your understanding of the material you have just studied and on which you will be tested. In fact, the data indicate that the most productive time for sleep in terms of your learning is the last few hours. Therefore, resist the urge to set an alarm to wake yourself up early: the snooze button is your friend on the morning before you are to write a test. Let the cock crow until he’s hoarse.
One study of a 2016 first year physics course for life science majors showed that students who slept less than 3 hours the night before the final exam had an exam score of \((50 \pm 5)\%\) while students who slept 7 to 9 hours the night before had a score of \((68 \pm 5)\%\). The difference is almost two full letter grades. The figure shows the data.

If you have an evening test and study for it in the morning, then taking a nap of an hour or so between your studying and taking the test has also been shown to increase how well you do.

In the previous sub-section you were urged to consider going for a walk just before the test. Another strategy, then, is to instead take a nap.

Earlier, we learned that breaking up your studying into two or three sessions separated by a few days is better than a single long session. The fact that you will sleep between the different sessions is one of the reasons why this strategy is effective.

“Chocolate is the answer. Who cares what the question is?” -- Anonymous

Earlier we discussed how a long period of high-level thinking can lead to a glucose deficiency in the brain, and when this happens you literally are not capable of high-level thought. When studying, eating something with a good glycemic index, like a protein bar, can partially compensate for this.

However, on a test you are often under time pressure, and it takes a while for the protein bar to be metabolised and produce glucose. If you eat or drink something with sugar in it, the glucose in the blood stream starts to be replenished in as little as 10 minutes. Note that sugar substitutes, like Splenda, will not work.

This suggests that, for example, taking some chocolate to the test or exam and eating it when you start to flag can be a useful strategy. But beware of the “sugar crash” which can lead to tiredness and lethargy.

**About Multiple-Choice Physics Tests**

So far our discussion of doing as well as possible applies to a test or exam in any course. In Physics, there is another issue about tests. Typically somewhat over half or more of the grades on each test are from multiple-choice questions. For a course that emphasises facts, students can answer a typical multiple-choice question in just one minute or so, which allows for many questions on the test, each of which is worth only a small grade. Physics emphasises concepts and applying them to solve problems. A typical multiple-choice question on a Physics test, then, takes about 5 minutes or more to do. This means
that a typical Physics test will have about one-fifth as many multiple-choice questions on it than a test in a course that is fact based, and each question will therefore have to count for more grades.

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I also thank Prof. Vincent Coletta, Loyola Marymount University, for permission to use the figure on page 11.

To Learn More

It could be worth your while to compare and contrast what I’ve written above with one or more of the documents listed below. How are they similar? Do they disagree with each other on anything?

- http://www.oberlin.edu/physics/dstyer/StudyTips.html This is a nice brief summary compiled by Dan Styer at Oberlin College near Cleveland, Ohio.
- http://www.physics.utoronto.ca/~jharlow/teaching/testhints.htm Another nice brief summary, this one by Jason Harlow, Dept. of Physics, Univ. of Toronto.

In addition, in the first section I mentioned Learning Communities in the context of forming or joining a study group. At the University of Toronto you can find such communities from the web site: http://flc.utoronto.ca/. For other schools, check your institution’s web site, or consult the partial database of such programs at: http://wacenter.evergreen.edu/lcdirectory/index.html.


The figure on page 2 has been around on my discs for so long that I don’t remember where I got it. If it is yours and you want it removed or a credit added, please contact me.

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