Perspectives on modern reformed pedagogy from the history of science, psychology, and the performing arts

David M. Harrison
Dept. of Physics, Univ. of Toronto
david.harrison@utoronto.ca

In part because of Physics Education Research, evidence-based teaching methods are becoming much more prevalent in physics courses in particular and STEM course in general at a variety of educational levels. This type of pedagogy is given various names, and here it will be called *interactive engagement*. This note examines the adoption of, and resistance to, interactive engagement pedagogy from the perspective of the history of science in the western world. Psychology will feature prominently in the discussion. The performing arts, notably improvisation and storytelling, will have contributions to make.

From the medieval period through the middle ages, natural philosophy (physics) and natural history (biology) were based on the thinking of authorities, notably Aristotle (384 – 322 BC) for natural philosophy and Galen (b 130 AD) for natural history. During this period, scholars seem to have been almost literally blind to evidence right in front of them that contradicted the pronouncements of these priest-like authorities. C.G. Jung believed that these blind spots indicated features of the *collective unconscious*, the instincts and archetypes shared by all humans, and he studied alchemy extensively to learn more about it.[1]

Our introductory physics texts, if they choose to give any historical information at all, typically mention how Kepler, Galileo, and especially Newton rejected the authority of Aristotle. Perhaps mentioned in this context will be the motto of the Royal Society of 1660: *Nullius in verba* (*Take nobody's word for it*). After Newton, according to this popular accounting, scientists became fully conscious[2] and rational beings, and the scientific method prevailed over "superstition". Alexander Pope expressed this view in his well-known epitaph for Newton (1727):

> Nature and Nature's Laws lay hid in Night:
> God said, "Let Newton be!" and all was light.

This, of course, is a vast over-simplification. Newton himself left over one million words of manuscript on his experiments and thinking about alchemy.[3]

Further insight into the fact that scientific materialism was and is more of an evolution than a revolution is from early 19th century biology. The life sciences were dominated by the view that living organisms contained some sort of "vital energy." Helmholtz was originally a physiologist, and in 1852, at age 21, he and three other physiologists signed an oath repudiating vitalism and resolving to consider only physicochemical forces.[4] They signed the oath in blood, like pirates. As Jaynes wrote, "This was the most coherent and shrill statement of scientific materialism up to that time. And [it was] enormously influential."[5] One of those strongly influenced by the oath was Freud. Five years after signing it Helmholtz proclaimed his Principle of Conservation of Energy.

In Ref. 2, Jaynes argues that human consciousness is a so-far incomplete transition from unquestioned acceptance of the authority of the gods or priests to a self-aware rationality. In such a view, it is plausible to argue that part of the resistance to Einstein's theories of relativity in the early 20th century was because some physicists, still subject to in an older pre-rational way of thinking, had elevated Newton to a status of infallibility. This, of course, is precisely the sort of thinking that the Royal Society's motto condemned. In any case, economist John Maynard Keynes seems to have somewhat overstated the case about the rationality of scientists after Newton when he said:

> Newton was not the first of the age of reason. He was the last of the magicians, the last of the Babylonians and Sumerians, the last great mind that looked out on the visible and intellectual world with the same eyes as those who began to build our intellectual inheritance rather less than 10,000 years ago.[6]

It is tempting to think that now, in the early 21st century, such irrational and mystical thought processes have disappeared, at least amongst scientists. So we dismiss as just stupid those anti-vaccination parents concerned about a link with autism who, when shown data that prove irrefutably that there is no link between vaccinations and autism, end up even more strongly anti-vaccination than before. Exposure to the data has caused the parent to elevate a dislike of vaccinations from a prejudice to a quasi-religious infallible principle. It is amusing to wonder if a Jungian analyst would find some aversion to the process of vaccination to be an aspect of the collective unconscious.

Surely you and I, the reader and writer of this note, are immune to the irrational behavior being shown by the anti-vax parents. Except that modern psychology has shown that these non-logical thought processes are universal.[7] *So now I am not so sure about you.*

Piaget provides a different but complementary view to Jaynes.[8] In Piagetian analysis, the cognitive development of young people consists of four stages:

1. Sensorimotor (birth – 2 years). Learns that he/she is separate from the external world; learns about object permanence.
2. Pre-operational (2 – 7 years). Can represent objects as symbols which can be thought of separately from the object; can "make believe;" wants the knowledge of knowing everything.
3. Concrete Operational (7 – 11 years). Can reason logically about concrete events or objects; acquires concepts of conservation of number, area, volume, and orientation.

4. Formal Operational (11 – 17 years and onwards). Can reason logically about abstract formal concepts; can reason with ratios; can do separation and control of variables; can think about different points of view or reference frames; can think about thinking.

The ability to use the ways of thinking, i.e. the operations, associated with Formal Operations is clearly necessary to do science in general and physics in particular. However, as Arnett wrote: "research has shown that not all persons in all cultures reach formal operations, and most people do not use formal operations in all aspects of their lives."[9] The similarities of these stages in the development of young people to the history of science are striking.[10] Educators have long found Piaget's taxonomy useful in thinking about effective pedagogy.[11]

As a person's thinking becomes more developed, some aspects of their thinking are almost prototypical. For example, a young child believes that their parents are infallible. As their cognition develops they realize that their parents are sometimes wrong about things, and in adolescence this can often become a conviction that their parents are wrong about *everything*. This is also a time when people seem particularly susceptible to accepting as the infallible "Key to the Universe" a religious prophet, political philosophy, a particular subculture, or even a psychedelic drug. Later many people mature to the point that they can look back and decide that maybe their parents were not so dumb after all.

A similar process occurs in physics. Physics teachers have long known that the students in their introductory courses were largely Aristotelian in their views about dynamics, and diagnostic instruments such as the Force Concept Inventory were originally developed to aid teachers in identifying and "correcting" these wrong views. For example, the Aristotelian view of motion is that except for gravity, a force is always necessary to cause motion. We try to convince our students that Aristotle was totally wrong about this, and that forces cause changes in the state of motion. Thus considerable effort is devoted in the classroom to forcing the students through the threshold and into the Light of the Newtonian view: an object in motion stays in motion with the same speed and direction unless a force causes it to change that motion. In the process, perhaps the teacher has harsh criticism for Aristotle and his stupidity. Perhaps later the physicist realizes that the 2nd Law of Thermodynamics insures that macroscopically the circumstance of no frictional forces whatsoever cannot exist anywhere in this universe. This means that in the real world, as opposed to an unattainable ideal one, to sustain an object in uniform motion always requires applying a "balancing force." Maybe Aristotle, like our parents, wasn't so dumb after all.

With this background perspective, we now consider the traditional lecture format for teaching. The origins are the medieval universities, when the lecturer was the only one with a copy of the book, and would read it to the students, perhaps with some added commentary. Figure 1 shows a lecture at the University of Bologna in the mid-fourteenth century.[12]

Figure 1. A mid-fourteenth century lecture at the University of Bologna

The similarities to a medieval priest commenting in the language of the parishioners on the Latin bible are striking. The lecture has persisted in many classrooms to the present day, although since Gutenberg, credited with inventing the printing press in about 1440, the students have the book too. Similarly although between the printing press and translations of the bible into other languages made its contents available to all literate people, the sermon has persisted. As we have seen, in medieval times and the middle ages, there was a nearly universal unquestioning acceptance of the authority and infallibility of the lecturer/priest, and apparently this tendency persists to this day.

There has been some resistance to lecturing as a pedagogical method: the example of Socrates is well known, and Galileo is often credited with saying "You cannot teach a man anything, you can only help him find it within himself."[13] Education research, perhaps especially Physics Education Research, has now shown conclusively that for most students the lecture is far inferior to interactive engagement methods of instruction. For example, Freeman et al. recently did a meta-analysis of 225 studies comparing lectures to interactive engagement in STEM courses and confirmed that interactive engagement was more effective than traditional lectures.[14] So it is initially mysterious that some science teachers, despite the evidence, continue to lecture to their students.[15] Surely science teachers are willing to accept empirical evidence, unlike the anti-vaccination parents.

The perspective of this note has been that part of the reason for the desire of students for a voice of authority and the desire of some teachers to be that voice are a vestige of an ancient way of thinking based on an unquestioning belief in the voice of the gods and/or their priests or kings. However, modern psychology, as described in Reference 7 and other sources[16], can offer some further insights.

To slightly oversimplify, we can classify human thinking as having two modes, often called *System 1* and *System 2*. System 1 is fast, automatic, effortless, and largely unconscious. System 2 is slow, logical, effortful and conscious.[17] System 2 is often taken to be evolutionarily recent,[18] which is reminiscent of Jaynes' analysis of the historically recent emergence of consciousness. We live most of our lives under the control of System 1, which is capable of allowing us to drive a car on an empty road, recognize whether another person is frowning or smiling, understand simple sentences, etc. An expert physics problem solver can often intuitively solve or at least outline how to solve a physics problem using System 1. When a situation is too complex for System 1, it invokes System 2. Examples include driving a car in a blizzard, looking for a woman with white hair in a crowd, multiplying 17×24 in your head, etc. Beginning physics students must use System 2 to solve most physics problems.

In Jaynes' analysis, people had no "free will" in ancient times: what is now called System 1 dictated what people would do. Homer's *Iliad* is a record of that way of "thinking" and in Jaynes' reading none of the human characters in the story were conscious. For Jaynes it is not accidental that Homer created the *Iliad* as an oral story that was only transcribed into a book a few centuries later. In fact, written records as more than a tool for bookkeepers only developed when the voices of System 1 began to go silent. In this context, it is interesting to note that it was only in the 10th century A.D. that reading became a normally silent activity: in antiquity it was normal to read aloud.[19] When Augustine (later Saint Augustine) visited Ambrose, his teacher, in 384 A.D. he was astounded to discover that Ambrose was reading a book but not saying anything. For Augustine and his contemporaries the spoken word was an intricate part of the text itself.

Jaynes wrote that, "Reading in the third millennium BC may therefore have been a matter of *hearing* the cuneiform, that is, hallucinating the speech from looking at picture-symbols, rather than visual reading of syllables in our sense." In Reference 18 Miguel suggests that, "This 'aural hallucination' may have been true also in the days of Augustine, when the words on the page did not just 'become' sounds as soon as the eye perceived them; they *were* sounds." An interesting question is: did the readers in the third millennium BC read aloud, or did that arise only when the voices of the aural hallucinations became quieter?

It is also interesting to note that there are primary level teachers who believe that when young people learn to read, it reduces their ability to remember[20]; of course the children are taught to read silently. Perhaps what is thought of as a causal link from learning to read silently to a loss of the ability to remember is instead a correlated manifestation of the young person's development into a higher stage of cognitive ability, similar to the historical transition from an earlier "bicameral" mind to modern consciousness.[21] Further, it is probably reasonable to think of Jung's work on the collective unconscious as an exploration of the cognition of what Jaynes called the bicameral mind and modern psychology calls System 1.

Because System 2 requires a comparatively large expenditure of energy and effort, it is typically invoked only when the individual is forced to do so. Realizing that this is true has implications for teachers. For example, when teaching, say, Newtonian mechanics for the second or more times, one hears the same or similar questions repeatedly from the students. So, the first time or two that the teacher has heard the question, she has probably been using System 2 to carefully listen to and parse what the student is saying. But after that, it is a natural tendency to believe after the first few words that she knows what the question will be, quits listening, and goes into automatic System 1 thinking about formulating the answer. As Arons repeatedly reminded us, often our failure to shut up and listen carefully to our students dooms us to inappropriate responses to their difficulties.[22]

People skilled in the art of improvisation have developed a number of "games" that assist people in learning to carefully listen to each other. Many of those games can be classified under the rubric of "YES, AND." An example, similar to one given by Tina Fey, is: [23]

> Speaker 1: " I can't believe it's so hot in here."
> Speaker 2: "**YES, AND** that can't be good for the wax figures."
> Speaker 1: "**YES, AND** the melted wax is making a mess on the floor."
> Speaker 2: "**YES, AND** …"

For the speakers to respond appropriately to each other, they must listen carefully. Institutions such as the Alan Alda Center for Communicating Science[24] exist to assist scientists, both researchers and teachers, in learning how to use these and other techniques in their communication.

Because System 1 is unconscious, we are usually unaware when we make decisions using it. For example, consider the following:[25]

> A bat and ball cost $1.10.
> The bat costs one dollar more than the ball.
> How much does the ball cost?

The number 10, i.e. 10¢, probably came to your mind without conscious thought. Perhaps that is your answer to the problem: more than one-half of undergraduate students give this answer. But invoking System 2 to do a simple check shows that this answer is wrong: if the ball cost 10¢ and the bat cost one dollar more, $1.10, then the bat and ball together cost $1.20. Since many or most of us are lazy, we don't do that check. However, we can force System 2 to deal with the problem by, for example, presenting it in a small font in washed-out gray print: System 2 is required to read and parse the question and, once invoked, goes on the solve the problem.[26] In this example the performance on the difficult-to-read version of the question increased dramatically over the version when it was easy to read.

Lacking such external triggers, we need to exert considerable self-discipline to force our conscious System 2 mind to deal with issues and questions. A personally somewhat embarrassing example regards the introductory physics course at the University of Toronto. The first half of a 2-semester sequence deals mostly with Newtonian mechanics. It is offered both in the Fall term with a normal 12-week term, and during the Summer with a compressed 6-week term. Those of us involved in teaching this course were convinced that the compressed format did not allow the students adequate time to reflect on and absorb the sometimes-difficult concepts of the course, and urged our administration to change the summer version to a normal 12-week format. The administration resisted, in part because with a 6-week term students could and did complete both halves of the course in a single summer. Therefore, we used normalised gains on the Force Concept Inventory to prove to the administration that the summer format should be changed. Except that the results were that the difference in educational effectiveness between the Fall and Summer terms was negligible.[27] The point is that, although between us we had nearly 100 years of teaching experience, our intuition was wrong. We immediately dropped our request to change the format of the course.

In ancient times, the answer to the bat and ball problem was in fact what infallible System 1 said it was: 10¢. Although "wrong", a moment's reflection may convince you that of all the possible answers in the universe this is one of the more reasonable ones. If you are buying the ball from a friend does it really matter whether you gave her 10¢ instead of 5¢? The point is that System 1 does a pretty good of job of quickly and effortlessly coming up with a pretty good answer to many of life's everyday problems. In solving the bat and ball problem to get the right answer of 5¢, a cognitive conflict was probably set up between your System 1 and your System 2. In ancient times System 2 was not yet capable of taking control. Today when there is a conflict between System 1 and System 2, as naturalist Stephen Jay Gould (1941-2002) wrote, System 1 is often reduced to "a little homunculus in my head [which] continues to jump up and down, shouting at me."[28]

In pre-conscious times, the way people understood the world was from the voices of the gods, priests, and kings, and from oral stories such as those later transcribed into the *Iliad*. This form of cognition persists today in the ability of System 1 to construct a causal narrative about a situation, which often is a non-logical extrapolation from only a few facts. For example: "Fred's parents arrived late. The caterers were expected soon. Fred was angry." You immediately know why Fred was angry, but a moment's consideration makes it clear that nowhere does a causal link from the parent's tardiness to Fred's anger appear anywhere other than as a construction of your own System 1 mind.[29] Our understanding of the world is deeply rooted in such narrative stories constructed and/or perceived by System 1. But many STEM teachers seem to have "storyphobia."

In our classes and general talks we usually present a nice, logical, System 2 argument to the students or audience: "Blah blah **AND** blah blah **AND** blah blah **AND THEREFORE** blah blah blah." One useful narrative structure is "Blah blah **AND** blah blah **BUT** blah blah **THEREFORE** blah blah blah." This *AND-BUT-THEREFORE* (ABT) structure is is used, for example, by South Park co-creator Trey Parker. Olson gives an example of the ABT structure in a well received talk titled "Sea Level Rise:

New, Certain and Everywhere." The storyline was: [30]

> Sea level was relatively stable for 8000 years
> **AND** coastal communities were built on the assumption of stability,
> **BUT** over the past 150 years the level has been rising.
> **THEREFORE** a new approach to coastline management is needed.

STEM teachers need to learn from the storytellers how to communicate to both System 1 as well as System 2 of our students. As Olson wrote: "everyone can and should incorporate narrative structure to their science communication endeavors."

Often wrong ideas are built as narrative structures in the person's thinking. It is plausible to argue that this is what is happening with the anti-vaccination parents, and we learn from their refusal to accept data that conflicts with their narrative that System 2 by itself is inadequate in changing their minds. Similarly for our students when we address their wrong ideas about the physical world: if we ignore their System 1, it will just jump up and down and shout inside their heads.

One of the common ways of implementing interactive engagement pedagogy is to replace lectures by collaborative teams of four or so students who work as a group on problems, questions, and concepts together. This form of interactive engagement pedagogy is proven to be effective.[31] The bat and ball problem allows for a "toy" example of how this can be implemented. When we invoked System 2 to check the intuitive answer of 10¢, it was probably immediately obvious that this answer is wrong. In a team of 4 the chances are very high that at least one of the members will realize that the answer is not 10¢, and so the entire team ends up leading itself to the correct answer. Note that in such a case, the instructor's only job would be to intervene if any of the teams under his supervision did not have a student member who realized that 10 is the wrong answer, and to guide the team to that realization: the team will then go on to the correct answer without further intervention. As we tell our graduate student instructors in Toronto, "Keep your hands in your pockets, figuratively if not literally."

There is a lot of research comparing experts to novices in how they solve physics problems.[32] Many introductory physics textbooks now present the results of this research into a specific step-by-step problem solving strategy. They all contain as the last step that the student should assess the result. Does it make sense? Are limiting cases (such as setting the acceleration due to gravity to zero) reasonable? We can view this advice as an explicit attempt to get the student to invoke System 2 as part of the final step of their problem solving.

It should be emphasized that System 1 is not stupid, just different from our conscious System 2 mind. Already mentioned is the fact that physics experts can use System 1 to solve or at least know how to solve a physics problem almost instantaneously. Similarly, a chess expert can glance at the board and intuitively know which moves will be most productive. So a question is: why is an experienced teacher's intuition about the correct way to interact with her students so often wrong when an experienced physics problem solver's intuition about how to solve a problem so often correct? Part of the answer may be that in solving a physics or chess problem, the feedback from the outcome of various strategies is received fairly soon after the strategy has been tried. In the case of education, the outcome of a particular intervention is usually not apparent for some days, weeks, months, or even years: this makes it difficult for System 1 to "learn" which techniques are most effective. A common and sad example is when a teacher gives a student a brilliant, fact-filled, and logical answer to a question, and at that moment the student actually knows the answer. However, when the student is tested for that understanding a few days or weeks later, it has evaporated.

The data are clear that the learning from using the methods of interactive engagement persists for days, weeks, and even years. The quick, intuitive System 1 conclusions, such as that the ball cost 10¢, are moderated by the interactions between students working as a team. As Kahneman wrote, "Organizations are better than individuals when it comes to avoiding errors, because they naturally think more slowly and have the power to impose orderly procedures."[33] Educators implementing collaborative interactive engagement pedagogy may wish to keep track of the rapid advances in our understanding of how to make group dynamics particularly effective.[34] It should also be emphasized that interactive engagement pedagogy needs to be built on a foundation of facts and ideas. Some time is required to provide this foundation: it can be delivered by the instructor in the classroom, and/or from the textbook, and/or from student interactions with an apparatus, and/or from a *YouTube* video.

In the context of the issues discussed here, it is interesting to note that three more-or-less independent psychological threads come to similar conclusions. Piaget (re-discovered by psychologists in the 1960s), Jaynes in 1976, and the modern Type 1 / Type 2 taxonomy developed in the past 40 years all posit that the ability for a rational, conscious, logical type of cognition is a relatively recent development in humans, and that the development of this capacity mirrors the development of science.

In conclusion, our intuitive System 1 thinking is an ancient way of cognition, and the scientific [r]evolution can be viewed as an example of the emergence of System 2, which is a different form of thinking. Both systems are useful and necessary in their proper place. But without external stimulus or self-discipline, we all tend to revert to System 1 thinking, which for people in general and educators in particular can lead to incorrect and potentially damaging conclusions. Thus to be an effective educator one needs to resist the urge to blindly accept one's intuition. At the same time, in our communication with our students, we should try to address both their intuitive and logical thought structures. Both of these teaching methods require hard work: as colleague John Pitre says, "If you want to be a better teacher, work harder!"

REFERENCES & NOTES

[1] See, for example, C.G. Jung, *Psychology and Alchemy*, which is Volume 12 of *The Collected Works of C.G. Jung,* (Princeton University Press, 1968).
[2] The word *conscious* in this context is as used by J. Jaynes, *The Origin of Consciousness in the Breakdown of the Bicameral Mind* (Houghton Mifflin, 1976).
[3] The University of Indiana has made Newton's work on alchemy available. See: http://www.newtonproject.sussex.ac.uk/prism.php?id=46 (Retrieved Jan. 8, 2016).
[4] The oath is now known as the Reymond-Brucke oath. See, for example, *The Encyclopedia of Human Thermodynamics* page at: http://www.eoht.info/page/Reymond-Brucke+oath (Retrieved Jan. 9, 2016).
[5] Ref. 2, pg. 437.
[6] J.M. Keynes, "Newton, the Man," a lecture to the Royal Society in 1946. Available at: http://www-history.mcs.st-and.ac.uk/Extras/Keynes_Newton.html (Retrieved Jan. 13. 2016).
[7] See, for example, D. Kahneman, *Thinking, Fast and Slow* (Farrar, Straus and Giroux, 2011).
[8] B. Inhelder and J. Piaget, *The Growth of Logical Thinking From Childhood to Adolescence: An Essay On The Construction of Formal Operational Structures* (Basic Books, 1958).
[9] J.J. Arnett, *Adolescence and Emerging Adulthood: A Cultural Approach*, 3rd ed. (Pearson, 2010), pg. 89.
[10] See, for example, C. Franco and D Colinvaux-De-Doninguez, "Genetic epistemology, history of science and science education," Science & Education **1**(3), 255 (1992).
[11] See, for example, D.M. Harrison, "Factors correlated with students' scientific reasoning ability in an introductory physics course," which is a chapter in a forthcoming book: K.A. MacLeod and T.G. Ryan, *The Physics Educator: Tacit Praxes and Untold Stories* (Common Ground Publishing). A pre-print is available at: http://www.upscale.utoronto.ca/PVB/Harrison/CTSR_Factors/CTSR_Factors.pdf
[12] Laurentius de Voltolina - The Yorck Project: *10.000 Meisterwerke der Malerei.* Downloaded from: https://en.wikipedia.org/wiki/Lecture - /media/File:Laurentius_de_Voltolina_001.jpg (Retrieved January 9, 2016).
[13] It is questionable whether Galileo really wrote this.
[14] S. Freeman et al., "Active learning increased student performance in science, engineering, and mathematics," PNAS **111**(23), 8410 (2014).
[15] Some further remarks on the mystery were explored in a Canadian Association of Physicists talk given on receiving the CAP gold medal for undergraduate education in 2012. It is available at: http://www.upscale.utoronto.ca/PVB/Harrison/ResistanceToPER/ResistanceToPER.pdf (Retrieved Jan. 9, 2016).
[16] For example, M. Gladwell, *Blink: The Power of Thinking Without Thinking* (Back Bay Books, 2005).
[17] The word *conscious* in this context is essentially identical to the way Jaynes used it in Ref. 2.

[18] S. Mithen, *The Cognitive Basis of Science* (Cambridge Univ. Press, 2002), pg. 23 ff.
[19] A. Manguel, *A History of Reading* (Penguin Books, 1996), pg. 42-43.
[20] Actually it was my mother, Mary Davis Hughes.
[21] A favorite study of memory is F. Yates, *The Art of Memory* (Routledge & Kegan Paul, 1996). Also available at:
monoskop.org/images/b/be/Yates_Frances_A_The_Art_of_Memory.pdf
(Retrieved Jan. 22, 2016). Of particular relevance for the topics of this note is the final Chapter XVII, "The Art of Memory and the Growth of the Scientific Method," pg. 268 ff.
[22] For example, A.B. Arons, *Teaching Introductory Physics* (Wiley, 1997).
[23] A lovely discussion of some aspects of improv., including YES, AND, is T. Fey, "Tina Fey's Rules of Improvisation That Will Change Your Life and Reduce Belly Fat,"
http://sostark.net/post/4965998605/tina-feys-rules-of-improvisation-that-will-change
(Retrived Jan. 25, 2016).
[24] http://www.centerforcommunicatingscience.org/alan-alda/
[25] This example and discussion are based on Ref. 7 pg. 44 – 45, and pg. 65.
[26] The correct answer is 5¢
[27] J.J.B. Harlow, D.M. Harrison, and E. Honig, "Compressed-format compared to regular-format in a first-year university physics course," Am. J. Phys. **83**(3), 272 (2015).
[28] Ref 7, pg. 159.
[29] This example is from Reference 7, pg. 74-75.
[30] R. Olson, "Narratively Speaking," Science **342**, 1168 (2013).
[31] For example N.D. Finkelstein and S.J. Pollock, "Replicating and understanding successful innovations: Implementing tutorials in introductory physics," Phys. Rev. ST PER **010101** (13), 010101-1 (2005).
[32] A somewhat dated review is D.P. Maloney, "Research on Problem Solving: Physics" in D.L. Gabel ed., *Handbook of Research on Science Teaching and Learning* (Macmillan, NY, 1994), pg. 327 – 354.
[33] Reference 7, pg. 417 – 418.
[34] For example A. Pentland, *Social Physics: How Good Ideas Spread – The Lessons from a New Science* (Penguin Press, 2014).

Last revision: August 4, 2016