PHY 353S - Electromagnetic Waves
Spring 1999
Problem Set 1 - due 5:00pm February, 9th

You may use any method - except cheating - to solve these problems
There are no trick questions here.
Some may require thought, but none - so far as I know - require magic for the solution

1. In order to perform quick conversions between the various unit systems for spectral calculations, a table like the following is required:

		TO					
		cm^{-1}	m^{-1}	μm	THz	eV	J
FROM	cm^{-1}	1					
	m^{-1}		1				
	μm			1			
	THz				1	X	
	eV					1	
	J						1

The idea is that to get from the unit on the left to the unit along the top you multiply by the unit in the square. E.g. to get from THz to eV, you multiply by X -- 1THz = XeV

However there is one unit in the above table for which a straightforward conversion is impossible.

i) Fill in the table as far as possible
ii) Identify the “rogue” unit and explain why it is incompatible with the rest of the units

2. i) How far does light travel in a vacuum in 1ns?.
ii) If light travels through a medium of (real) refractive index n and thickness t. What is the time delay between a wave which has traversed the medium compared to one which has not?

3. i) Show that for a dielectric material, if we are far from any resonance and therefore κ (the absorption term) is small, an equation for the refractive index is:

$$(n^2 - 1)^{-1} = \lambda_p^2 / \lambda_0^2 - \lambda_p^2 / \lambda^2$$

(You will have to explain what the symbols mean)

ii) Augustin Louis Cauchy (1789-1857) came up with an empirical formula for the refractive index of materials as:

$$n = C_1 + C_2 / \lambda^2 + C_4 / \lambda^4 + \ldots$$

Can you explain how that relates to the formula above? (Cauchy was not familiar with materials of high refractive index - they hadn’t been discovered yet.)

iii) We assume that Cauchy’s formula is applicable in the visible. Try fitting the data for several of the glasses shown in the lecture to a two-term (ie with C_1 and C_2 only) Cauchy formula and see how good it is. Here’s the table I used for the graph that is in the lecture notes

Frequency (THz)	Zinc Crown	High Disp. Crown	light flint	heavy flint	heaviest flint
831.02	1.539	1.546	1.614	1.705	
691.24	1.528	1.533	1.594	1.675	1.945
617.28	1.523	1.527	1.585	1.664	1.919
509.34	1.517	1.52	1.575	1.65	1.89
457.32	1.514	1.517	1.571	1.644	1.879
390.63	1.511	1.514	1.567	1.638	1.867
250.00	1.505	1.507	1.559	1.628	1.848
150.00	1.497	1.497	1.549	1.617	1.832

4. If

$E_x = 0$, $E_y = A \sin(\omega t + kz)$, $E_z = 0$
$H_x = B \sin(\omega t + kz)$, $H_y = 0$, $H_z = 0$

i) Show that Maxwell's equations are satisfied by the above fields.
ii) Find the relationship between A and B
iii) In which direction does the wave travel?

5. Light is refracted and reflected by a smooth water surface

i) At what incident angle is the reflected beam totally plane polarised?
ii) What is the angle of incidence from below at which the energy is first totally reflected back down

6. Unpolarised light of intensity I, is sent through three linear polarisers arranged at angles of 0^o, 20^o and 40^o with respect to some arbitrary axis perpendicular to the direction of propagation. Using matrix methods, find the intensity of the final beam and it's direction of polarisation.

7. A birefringent material has two axes - the "privileged directions" - also called the "fast" and the "slow" axes. For a particular material which we will consider here, these privileged directions are mutually perpendicular and the indices of refraction of the material are 1.350 and 1.355 along these directions. We are trying to make a device to convert linearly polarised light into circularly polarised light.

i) What should the geometry of the incident beam of linearly polarised light be with respect to the privileged directions?
ii) What is the minimum thickness of the plate to make the conversion at 635nm?
iii) In practice these plates have to be a certain minimum thickness in order to be usable - otherwise they break too easily. If the minimum allowed thickness for handling is 1mm - what is the new minimum plate thickness for the linear-circular conversion?
iv) In order for the conversion to be within 1^o over the entire plane wavefront, what flatness is required from the plate? Express your answer in wavelengths of 635nm radiation.

8. Some liquids, including solutions of sugar, rotate the plane of polarisation of a linearly polarised beam as it propagates. For solutions of moderate strength, the rotation is proportional to the concentration. For cane sugar the rotation is 6.67^o/cm for a solution concentration of 1gm/ml. The device for making this measurement is called a saccharimeter. If the length of propagation is 9.82cm and the rotation is 8.24 ± 0.01^o, calculate the concentration (with errors) of the sugar solution.

9. A plane polarised beam of 5mW total power from a helium-neon laser is of Gaussian form $E = E_o \exp(-r^2/r_o^2)$ with $r_o = 0.3$mm. Calculate the value of E_o.

10. A beam of light is incident upon a parallel-sided glass plate of index of refraction 1.4 at Brewster's angle.

i) Show that if the first interface (air-glass) is at Brewster's angle, then the second interface is also at Brewster's angle.
ii) If the beam going into the plate is unpolarised, find the ratio of the intensities of the two principal planes of polarisation with respect to the plate after transmission through the plate ignoring multiple reflections
iii) Repeat ii) above but include multiple reflections
iv) How many of these plates must I put in parallel to produce a beam which is 99% polarised?