Lecture 16

Fabry-Perot and Thin Films

Fabry-Perot Interferometer

- F-P Interferometer consists of two (very) flat parallel plates, often reflectively coated, at a separation of d
- If incoming beam is at angle θ, the refractive index between the plates n and the radiation at wavelength λ,
- Phase delay is: $\delta = 2kd\cos\theta = (4\pi n/\lambda)\, d \cos\theta$
- If amplitude reflectivity is Γ and transmissivity is Φ
  - Could be complex if there are phase changes
- Multiple beams add up $E_0(\Phi^2 + \Phi^2\Gamma^2 e^{i\delta} + \Phi^2\Gamma^4 e^{i2\delta} + ...)$
- Result is $E_0\Phi^2/(1 - \Gamma^2 e^{i\delta})$ in intensity $I_0|\Phi^2|^2 /|(1 - \Gamma^2 e^{i\delta})|^2$
- If Γ is approx 1.0 (highly reflective) then we get some interesting issues

Fabry-Perot Interferometer

- Expanding

$$I = I_0 \frac{T^2}{(1-R)^2} \frac{1}{1 + \left(\frac{4R}{(1-R)^2}\right) \sin^2(\delta/2)}$$

- Evidently the quantity $4R/(1-R)^2$ is important
- Let $F^2 = \pi^2 R/(1-R)^2$ - F is the "finesse" of the F-P etalon
  - F is in the range 0 (bad) to infinity (wonderful, but useless)
  - If we assume that R + T = 1

$$I = \frac{I_{max}}{1 + \frac{4}{\pi^2} F^2 \sin^2(\delta/2)}$$

Fabry-Perot Interferometer

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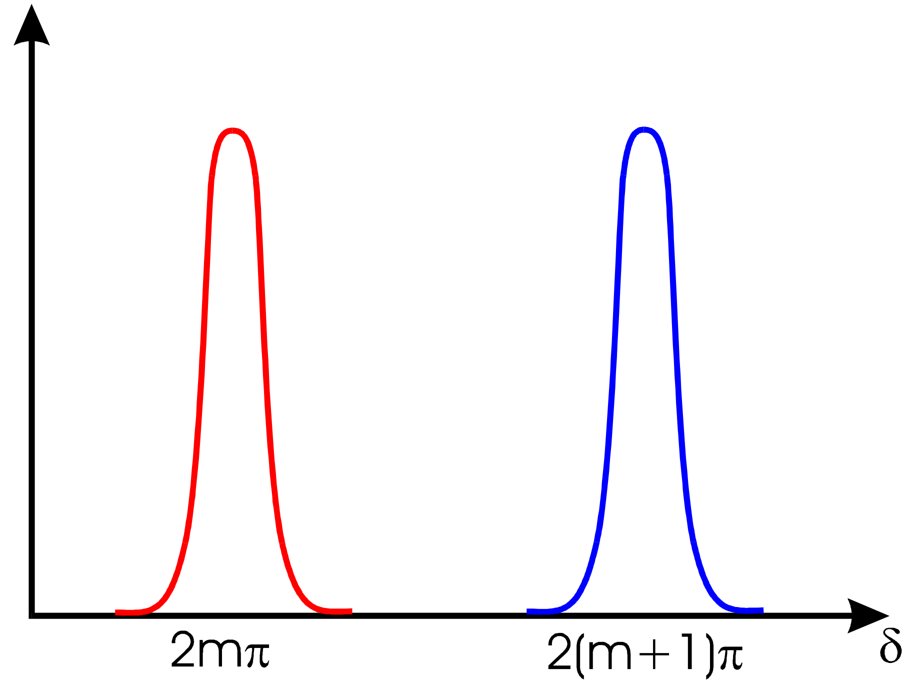


- Pattern repeats at $\delta = 2m\pi$
  - $\Delta\omega = \pi c/(n\ d \cos\theta)$
  - The "free spectral range" (fsr)
- Width of maximum is given by $4F^2\sin^2(\Delta\delta/2)/\pi^2 = 1$
  - If F large, $\Delta\delta = \pi/F$ or $\delta\omega = \Delta\omega/(2F)$

F-P Interferometer - Resolving Power

- Resolving power is $\lambda/\Delta\lambda$, but for large values can also be written as $\omega/\delta\omega$
- RP = $\omega/\delta\omega = 2F\omega/(fsr)$ (actually it's more nearly $F\omega/(fsr)$)
- Insert numbers
  - $F^2 = \pi R^2/(1-R)^2$, F = 175 for R = 0.99
  - $\omega = 1.885 \times 10^{15}$ rad sec^{-1} at 1µm
  - fsr = c/d at normal incidence for air cavity
    - = 3×10^8 sec^{-1} at 1000mm cavity length
  - RP = 10^9 (more or less) (300kHz at 300THz)
- For an FTI, D = 1000mm and λ = 1µm the resolving power is about 10^6
- For a grating spectrometer the resolving power is more like 10^4
- For a prism spectrometer more like 10^2

Very Small F-Ps - Thin Films

- Very thin film on a surface
- Film is parallel-sided
- Match electric field vectors at the surface
  - Need to distinguish p- and s- waves
- At the first surface
  - Incoming wave above interface
  - **Reflected wave above interface**
  - **Transmitted wave below interface**
  - **Reflected wave (from below) below interface**
- At the second surface
  - **Incoming wave above interface**
  - **Reflected wave above interface**
  - **Transmitted wave below interface**
  - Reflected wave (from below) below interface
- At each boundary we require tangential electric (E) and magnetic (H) fields to match above and below boundary

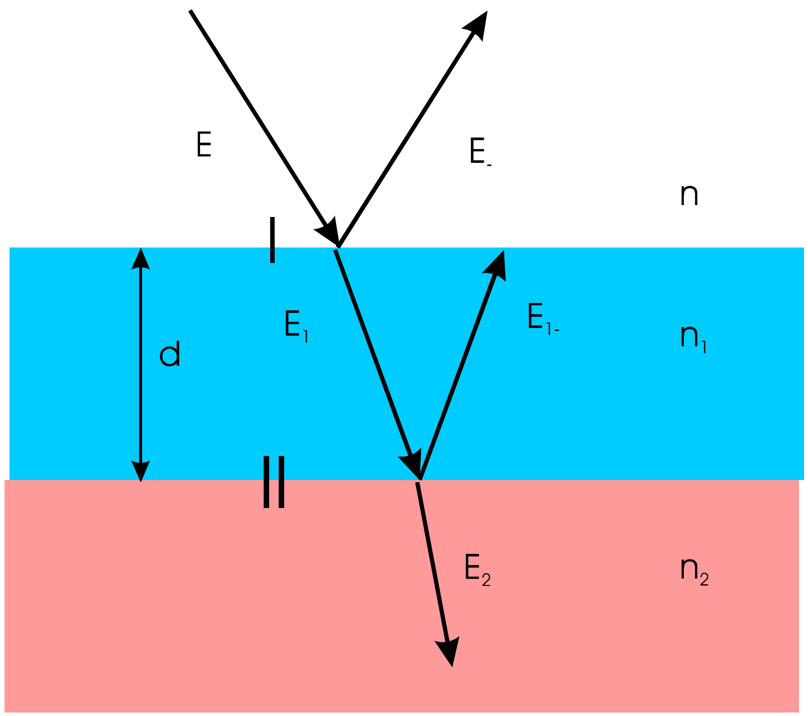
Thin Films - Single Layer

- For a single layer can neglect reflected field from below second interface
- Fields below first interface and above second are related by phase delay across the gap $\delta = kd\cos\theta = (2\pi n_1/\lambda)\ d\cos\theta$
- Work with an s-wave (TE mode)
- Match the fields at I and II

Thin Films - Single Layer

- $E_I = E + E_- = E_1 + E_{1-}$
- $H_I = (\varepsilon_0/\mu_0)^{1/2} (E - E_-) n\cos\theta = (\varepsilon_0/\mu_0)^{1/2} (E_1 - E_{1-}) n_1\cos\theta_1$
- $E_{II} = E_1 e^{i\delta} + E_{1-} e^{-i\delta} = E_2$
- $H_{II} = (\varepsilon_0/\mu_0)^{1/2} (E_1 e^{i\delta} - E_{1-} e^{-i\delta}) n_1\cos\theta_1 = (\varepsilon_0/\mu_0)^{1/2} E_2 n_2\cos\theta_2$
- Let $Y_1 = (\varepsilon_0/\mu_0)^{1/2} n_1 \cos\theta_1$
- Where Y_1 is the characteristic admittance of the medium

$$\begin{bmatrix} E_I \\ H_I \end{bmatrix} = \begin{bmatrix} \cos\delta & -i\sin\delta/Y_1 \\ -Y_1 i\sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} E_{II} \\ H_{II} \end{bmatrix}$$



- If we can write the matrix for one interface we can write it for many by concatenating matrices (in the right order!)

Thin Films - Multiple Layer

$$\begin{bmatrix} E_I \\ H_I \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} E_F \\ H_F \end{bmatrix}$$

- Now substitute explicitly for the first layer and the final layer

$$\begin{bmatrix} E + E_- \\ (E - E_-)Y \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} E_F \\ E_F Y_F \end{bmatrix}$$

- but $E_f = \Phi E$ and $E_- = \Gamma E$
- solve for Γ and Φ

$$R = \Gamma\Gamma^* = \frac{n_1^2(n - n_s)^2\cos^2\delta + (nn_s - n_1^2)^2\sin^2\delta}{n_1^2(n + n_s)^2\cos^2\delta + (nn_s + n_1^2)^2\sin^2\delta}$$

Thin Films - Single Layer

- Which is erudite - but we need the intensity which is $\Gamma\Gamma^*$.
- For a single layer (again) we know the terms
- if we choose $\delta = (2m+1)\pi/2$

$$R = \frac{(nn_s - n_1^2)^2}{(nn_s + n_1^2)^2}$$

$$\Gamma = \frac{Ym_{11} + YY_s m_{12} - m_{21} - Y_s m_{22}}{Ym_{11} + YY_s m_{12} + m_{21} + Y_s m_{22}}$$

- which is zero if $n_1 = (n_s n)^{1/2}$
  - Can be done with the right materials
  - anti-reflection coating

Thin Films - Single Layer

- For a single layer (again) we know the terms

$$R = \Gamma\Gamma^* = \frac{n_1^2(n - n_s)^2\cos^2\delta + (nn_s - n_1^2)^2\sin^2\delta}{n_1^2(n + n_s)^2\cos^2\delta + (nn_s + n_1^2)^2\sin^2\delta}$$

- if we choose δ = mπ

$$R = \frac{(n - n_s)^2}{(n + n_s)^2}$$

- Which is the same as if the coating were not there!!

Thin Films - Very Many Layers

- With more layers - can do more things
- Can make
  - transmissive
  - reflective
  - band-pass
- Must watch out that the TE and TM modes are different and therefore they will be polarising unless you watch out...