Ray Tracing
Casting the Problem for a Computer

Optical Ray Tracing

- Assumes that light propagates as rays in straight lines
- Processes permitted - in order of popularity
  - reflection
  - refraction
  - attenuation
  - polarisation
- Sign conventions
  - Many and mysterious
  - Use common sense and draw a diagram!!

Reflection at Plane Surfaces

- Every reflection at a plane surface reverses a component of the vector \( \mathbf{k} \)
- Three orthogonal reflections reverse all three components \( \mathbf{k} \rightarrow - \mathbf{k} \)
- A “corner-cube” reflector of three orthogonal mirrors always reverses the beam

Prisms are Fun!

- Prisms have plane surfaces, not necessarily orthogonal
- Prisms do two things
  - Refract the beam (dispersively)
  - Reflect the beam (coated or above critical angle)
- Every Reflection reverses a component of \( \mathbf{k} \)
- “reflects” the image in one dimension
Ray Deviation By Prism

- A simple case of Snell’s Law
- Angle of deviation $\delta$ given by
  - $\delta = \theta + \sin^{-1}[\sin\alpha(n^2 - \sin^2\theta)^{1/2} - \sin\theta\cos\alpha] - \alpha$
  - where $\alpha$ is the angle between the two prism faces
  - and $\theta$ is the angle ray makes with normal to 1st face
- Minimum deviation angle $\delta = 2\theta - \alpha$

(Symmetrical passage)

Refraction at Curved Surfaces

- Curved surfaces are >99% of the time spherical
- Once you go away from spherical, what do you use?
- Spheres have only one parameter (radius)
- Other conics have more
- Fictions employed for sanity (in order of popularity)
  - Rotational symmetry
  - All surfaces are spherical
  - All the rays cross the axis
  - Thin lenses
  - Paraxial Rays

Approximations

- Rotational symmetry
  - all optical components are circular
- All surfaces are symmetrical
  - Once you go away from spherical, what do you use?
- All the rays cross the axis
  - No “skew” rays
  - Rays can be characterised by where they cross the axis and a slope

Approximations

- Thin lenses
  - lens is so thin that thickness and curvature can be neglected
- Rays impact surfaces at same axial distance for all radial distances
- Paraxial Rays
  - All angles so small that $\tan\theta = \sin\theta = \theta$, $\cos\theta = 1$
Paraxial Forms

- Snell's law in paraxial, symmetric form
  \[ \frac{n_1}{s_1} = \frac{n_2}{s_2} \]
  - \( s_1, s_2 \) are the distances from the surface to the intersection of the ray and the axis
- For a spherical surface ROC R
  \[ \frac{n_1}{s_1} + \frac{n_2}{s_2} = \frac{n_2 - n_1}{R} \]

For a lens formed of two such surfaces

- \[ \frac{n_1}{s_1} + \frac{n_2}{s_2} = \frac{n_2 - n_1}{R_1} \]
- \[ -\frac{n_2}{s_2} + \frac{n_1}{s_3} = \frac{n_1 - n_2}{R_2} \]
- \[ \frac{1}{s_1} + \frac{1}{s_2} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{1}{f} \]

Good stuff - but limited!!

Ray Tracing by Computer

- Computers are stupid! (They do what you ask)
- First problem is to describe surfaces and rays
- Surfaces can be described in terms of equations
  \[ (x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2 \] (sphere)
  \[ z = z_0 \] (x-y plane)
- Need refractive index each side of surface
  - unless it's a reflector
- Rays as a position and the direction cosines
  \[ (x_0 + \lambda d_x) \hat{i} + (y_0 + \lambda d_y) \hat{j} + (z_0 + \lambda d_z) \hat{k} \]
  \[ d_x^2 + d_y^2 + d_z^2 = 1 \]
  \[ C = x + \lambda d \]
- Every ray requires 6 parameters - 3 position, 3 directions

Refraction
Ray Tracing by Computer

- Now need to describe the process
- Know how to do that if ray is
  - in a plane (eg x-z plane) tangentially normal to the
    surface (eg x-y plane)
- Intercept with surface is at the origin
- Easy stuff - but that’s not what we have!

Ray Tracing by Computer

- Locate intersection of ray \( C \) and surface \( p \)
- Often need to determine if
  - there is an intersection
  - which of two is needed
- Locate normal to surface \( n \) (all unit vectors, directions)
- Have incoming ray \( C = p_0 + \lambda c \)
- Apply Snell’s Law and derive new ray
  - Know that \( c.n = -\cos\theta \) - gives \( \theta \)
  - Use Snell’s law for \( \theta \)
  - If the outgoing ray is \( r \) then we also know \( n.r = -\cos\theta \)
  - We also know that \( c,n \) and \( r \) are co-planar
    - \( c x n = n x r \)
    - so can write \( r = a c + b n \)
  - Solve the equations for \( r \) - the refracted ray
- Actual ray path is \( R = p + \lambda r \)

Ray Tracing by Computer

- \( c.n = -\cos\theta \) - gives \( \theta \)
- Use Snell’s law for \( \theta \)
- If the outgoing ray is \( r \) then we also know \( r.n = -\cos\theta \)
- We also know that \( c,n \) and \( r \) are co-planar
  - \( c x n = n x r \)
  - so can write \( r = a c + b n \)
- \( r.r = 1 = a c.n + b n.r = a \cos(\theta, -\theta) - b \cos\theta \)
- \( r.n = -\cos\theta_r = a c.n + b = a -\cos\theta + b \)
- \( r = (\sin\theta, c + \sin(\theta, -\theta)n)/\sin\theta \)

Summary

- Given surface equation and ray equation \( S, C \)
- Compute point of intersection (get the right one) \( p \)
- Compute normal to surface at point of intersection \( n \)
- Apply Snell’s Law
- Have new ray \( R \)
- Repeat “ad nauseam” - Have to be computer for this!