Lecture 14

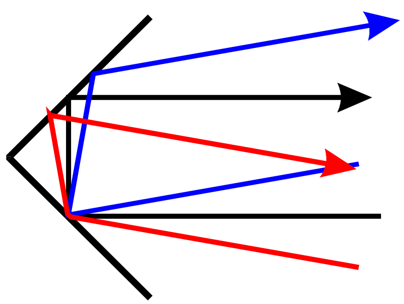
Ray Tracing
Casting the Problem for a Computer

Optical Ray Tracing

- Assumes that light propagates as rays in straight lines
- Processes permitted - in order of popularity
  - reflection
  - refraction
  - attenuation
  - polarisation
- Sign conventions
  - Many and mysterious
  - Use common sense and draw a diagram!!

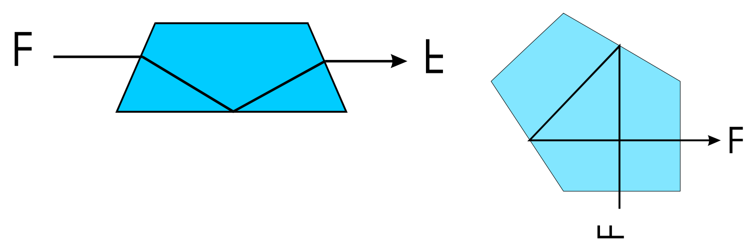
Reflection at Plane Surfaces

- Every reflection at a plane surface reverses a component of the vector **k**
- Three orthogonal reflections reverse all three components **k** -> - **k**
- A "corner-cube" reflector of three orthogonal mirrors always reverses the beam



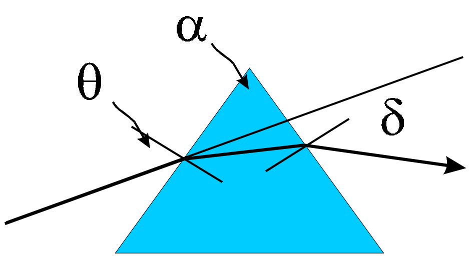
Prisms are Fun!

- Prisms have plane surfaces, not necessarily orthogonal
  - Prisms do two things
    - Refract the beam (dispersively)
    - Reflect the beam (coated or above critical angle)
- Every Reflection reverses a component of **k**
  - "reflects" the image in one dimension



Ray Deviation By Prism

- A simple case of Snell's Law
- Angle of deviation δ given by
  - $\delta = \theta + \sin^{-1}[\ \sin\alpha(n^2 - \sin^2\theta)^{1/2} - \sin\theta\cos\alpha] - \alpha$
  - where α is the angle between the two prism faces
  - and θ is the angle ray makes with normal to 1st face
- Minimum deviation angle $\delta = 2\theta - \alpha$ (Symmetrical passage)



Refraction at Curved Surfaces

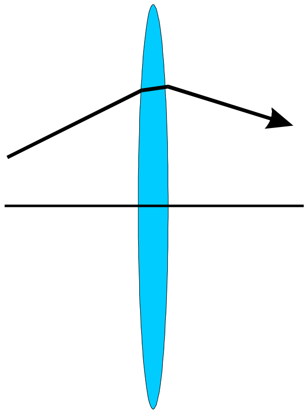
- Curved surfaces are >99% of the time spherical
  - Once you go away from spherical, what do you use?
  - Spheres have only one parameter (radius)
  - Other conics have more
- Fictions employed for sanity (in order of popularity)
  - Rotational symmetry
  - All surfaces are spherical
  - All the rays cross the axis
  - Thin lenses
  - Paraxial Rays

Approximations

- Rotational symmetry
  - all optical components are circular
- All surfaces are symmetrical
  - Once you go away from spherical, what do you use?
- All the rays cross the axis
  - No "skew" rays
  - Rays can be characterised by where they cross the axis and a slope

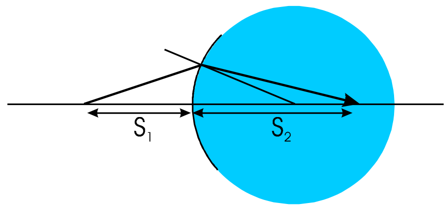
Approximations

- Thin lenses
  - lens is so thin that thickness and curvature can be neglected
  - Rays impact surfaces at same axial distance for all radial distances
- Paraxial Rays
  - All angles so small that $\tan\theta=\sin\theta=\theta$, $\cos\theta=1$



Paraxial Forms

- Snell's law in paraxial, symmetric form
  - $n_1/s_1 = n_2/s_2$
  - s_1, s_2 are the distances from the surface to the intersection of the ray and the axis
- For a spherical surface ROC R
  - $n_1/s_1 + n_2/s_2 = (n_2 - n_1)/R$



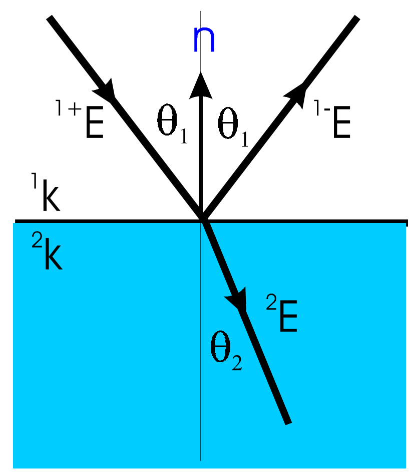
Paraxial Forms

- For a lens formed of two such surfaces
  - $n_1/s_1 + n_2/s_2 = (n_2 - n_1)/R_1$
  - $-n_2/s_2 + n_1/s_3 = (n_1 - n_2)/R_2$
  - $1/s_1 + 1/s_2 = (n_2/n_1 - 1)(1/R_1 + 1/R_2) = 1/f$
- Good stuff - but limited!!

Ray Tracing by Computer

- Computers are stupid! (They do what you ask)
- First problem is to describe surfaces and rays
- Surfaces can be described in terms of equations
  - $(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2$ (sphere)
  - $z = z_0$ (x-y plane)
  - Need refractive index each side of surface
    - unless it's a reflector
- Rays as a position and the direction cosines
  - $(x_0 + \lambda d_x)\,\mathbf{i} + (y_0 + \lambda d_y)\,\mathbf{j} + (z_0 + \lambda d_z)\,\mathbf{k}$
  - $d_x^2 + d_y^2 + d_z^2 = 1$
  - $\mathbf{C} = \mathbf{x} + \lambda\,\mathbf{d}$
- Every ray requires 6 parameters - 3 position, 3 directions

Refraction



Ray Tracing by Computer

- Now need to describe the process
  - Know how to do that if ray is
    - in a plane (eg x-z plane) tangentially normal to the surface (eg x-y plane)
    - Intercept with surface is at the origin
    - Easy stuff - but that's not what we have!!

Ray Tracing by Computer

- Locate intersection of ray **C** and surface **p**
  - Often need to determine if
    - there is an intersection
    - which of two is needed
- Locate normal to surface **n** (all unit vectors, directions)
- Have incoming ray $\mathbf{C} = \mathbf{p_0} + \lambda\mathbf{c}$
- Apply Snell's Law and derive new ray
  - Know that $\mathbf{c}.\mathbf{n} = -\cos\theta_i$ - gives θ_i
  - Use Snell's law for θ_r
  - If the outgoing ray is **r** then we also know $\mathbf{n}.\mathbf{r} = -\cos\theta_r$
  - We also know that **c**,**n** and **r** are co-planar
    - $\mathbf{c}\mathrm{x}\mathbf{n} = \mathbf{n}\mathrm{x}\mathbf{r}$
    - so can write $\mathbf{r} = a\mathbf{c} + b\mathbf{n}$
  - Solve the equations for **r** - the refracted ray
- Actual ray path is $\mathbf{R} = \mathbf{p} + \lambda\mathbf{r}$

Ray Tracing by Computer

- $\mathbf{c}.\mathbf{n} = -\cos\theta_i$ - gives θ_i
- Use Snell's law for θ_r
- If the outgoing ray is **r** then we also know $\mathbf{r}.\mathbf{n} = -\cos\theta_r$
- We also know that **c**,**n** and **r** are co-planar
  - $\mathbf{c}\mathrm{x}\mathbf{n} = \mathbf{n}\mathrm{x}\mathbf{r}$
  - so can write $\mathbf{r} = a\mathbf{c} + b\mathbf{n}$
- $\mathbf{r}.\mathbf{r} = 1 = a\,\mathbf{c}.\mathbf{n} + b\,\mathbf{n}.\mathbf{r} = a\cos(\theta_i - \theta_r) - b\cos\theta_r$
- $\mathbf{r}.\mathbf{n} = -\cos\theta_r = a\,\mathbf{c}.\mathbf{n} + b = a\,-\cos\theta_i + b$
- $\mathbf{r} = (\sin\theta_r\,\mathbf{c} + \sin(\theta_r - \theta_i)\mathbf{n})/\sin\theta_i$

Summary

- Given surface equation and ray equation **S**, **C**
- Compute point of intersection (get the right one) **p**
- Compute normal to surface at point of intersection **n**
- Apply Snell's Law
- Have new ray **R**
- Repeat "ad nauseam" - Have to be computer for this!