Lecture 13

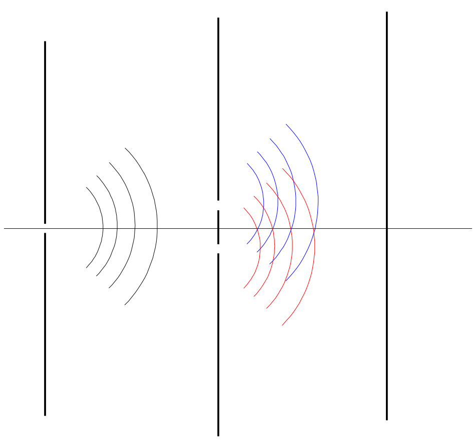
Coherence and Interference

Interference

- Interference can only occur if the two beams have the same frequency and therefore a defined phase
- If two beams are represented by
  - $\mathbf{E_1} = E_{01} \exp(i\mathbf{k_1}.\mathbf{r} - i\omega t)$
  - $\mathbf{E_2} = E_{02} \exp(i\mathbf{k_2}.\mathbf{r} - i\omega t + i\delta)$
- Then the intensity is
  - $I = |E_t|^2 = \mathbf{E_t}.\mathbf{E_t^*} = |E_{01}|^2 + |E_{02}|^2 + 2E_{01}E_{02}\cos\phi$
  - where $\phi = (\mathbf{k_1} - \mathbf{k_2}).\mathbf{r} - \delta$
- The last term is the interference term and says that the total intensity is not necessarily the sum of the individual intensities and in fact may be larger or smaller
- What is wrong with this expression for intensity?

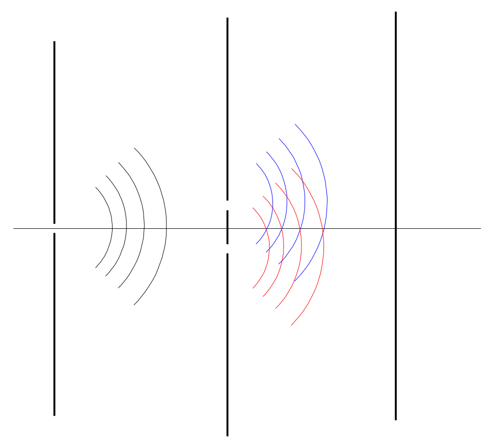
Young's Slits Revisited

- Why do the beams spread out? - Can't be plane waves
- Slit separation is d
- Distance to screen is z
- $\mathbf{k_1} - \mathbf{k_2} \cong \mathbf{y}\, kd/z$
- We assume that all angles are small, then
- $(\mathbf{k_1} - \mathbf{k_2}).\mathbf{r} = kyd/z$
- Intensity
  $I = 2I_0(1 + \cos(kyd/z))$

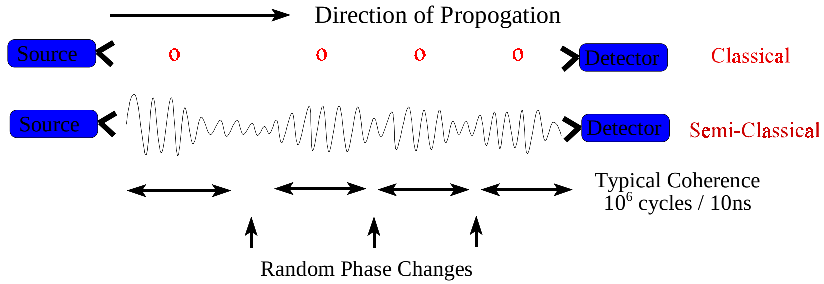


Young's Slits Revisited

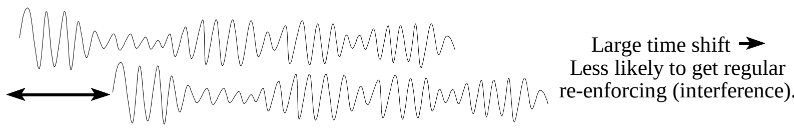
- Fringes die out for large angles/path differences
- Phase is not maintained over a long distance/time
- "Coherence length/time"
- Need additional notion of "partial coherence".
- We actually measure the time average of the fields which is the superposition of lots of "wavelets"
- $I = I_1 + I_2 + 2\mathrm{Re}\langle E_1^*.E_2\rangle$



Conceptual View of Coherence



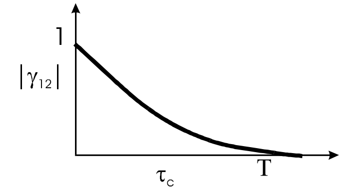
Combine two E-waves from same source:



- Hence Re<$E_1^*.E_2$> tends to zero over time.

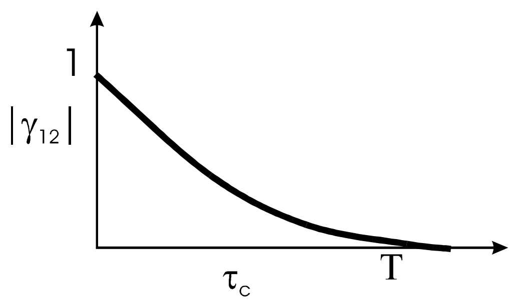
Correlation Function

- If the two waves originate from the same source - the interference can be considered to be in time
- $\Gamma_{12}(\tau) = \langle E_1^*(t).E_2(t+\tau)\rangle$
  - defines the "mutual coherence"
  - or the "correlation function"
  - $\Gamma_{11}(0) \propto I_1$
  - $\Gamma_{11}(\infty) \rightarrow 0$
- Normalised coherence function [Remove source intensities]
  - $\gamma_{12}(\tau) = \Gamma_{12}(\tau)/\sqrt{(\Gamma_{11}(0)\Gamma_{22}(0))}$
  - Time averaged intensity = $I_1 + I_2 + 2\sqrt{(I_1I_2)}\,\mathrm{Re}[\gamma_{12}(\tau)]$
  - $|\gamma_{12}(\tau)| = 1$ complete coherence
  - $0 < |\gamma_{12}(\tau)| < 1$ partial coherence
  - $|\gamma_{12}(\tau)| = 0$ complete incoherence $\{\tau \neq 0\}$



Correlation Function

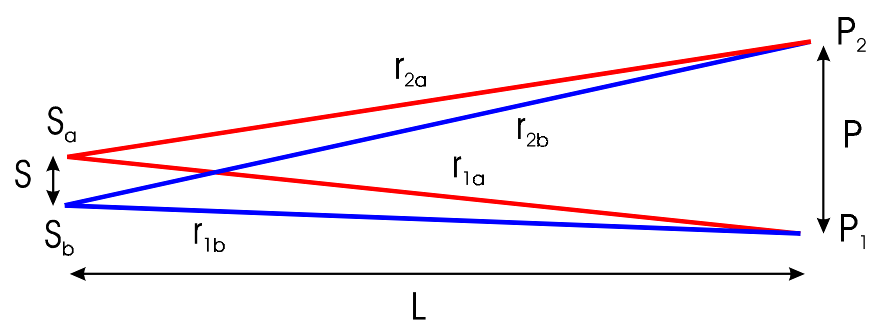
- Correlation decays with characteristic time τ_c
- Form of $|\gamma_{12}(\tau)|$ is often like an exponential decay



- Auto-correlation function (also known as self-correlation function) is linked to power spectrum P(ω) through fourier transform - the Wiener-Khintchine Theorem
- $\mathscr{F}(\Gamma_{11}(\tau)) = P(\omega)d\omega$
- Consider monochromatic source...
- This relationship is important for interferometers

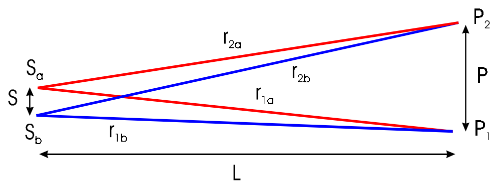
Spatial and Temporal Correlation

- Temporal coherence
  - Time is the only variable
  - Length is only used along the direction of propagation
- Spatial coherence
  - Links two space points, not on DOP
  - One point is generally fixed.
- Consider the following situation



Spatial and Temporal Correlation

- Spatial coherence between two points is defined as:



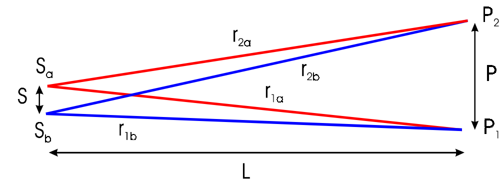
- $$\gamma_{12}(\tau) = \frac{\langle E_1^*(t)\ E_2(t+\tau)\rangle}{\sqrt{I_1 I_2}}$$

Each point has two components to E, BUT the sources are mutually incoherent so the cross-terms disappear in the average

$$\gamma_{12}(\tau) = \frac{\langle E_{1a}^*(t)\ E_{2a}(t+\tau)\rangle}{\sqrt{I_1 I_2}} + \frac{\langle E_{1b}^*(t)\ E_{2b}(t+\tau)\rangle}{\sqrt{I_1 I_2}}$$

Spatial and Temporal Correlation

- If the sources are point, monochromatic, then each term is almost a self-coherence function



- Common form for a self-coherence function is:

$$\gamma(\tau) = \exp(-i\omega\tau)\ \exp(-|\tau|/\tau_0)$$

- where the τ is just the path difference converted to a time $\tau_x = (r_{xa}-r_{xb})/c$
- If we assume vertical distances >> horizontal we find that $\tau_a - \tau_b = SP/(cL)$
- Write out the full coherence function as:

$$\exp(-i\omega\tau_a)\ \exp(-|\tau_a|/\tau_0)$$
$$(1 + \exp(-i\omega(\tau_b - \tau_a))\ \exp(-(|\tau_b| - |\tau_a|)/\tau_0)/2$$

Spatial and Temporal Correlation

- Taking the modulus and making the appropriate substitutions

$$|\gamma_{12}(\tau)| = \frac{1 + \cos\left(\frac{\omega SP}{cL}\right)}{2}\ \exp\left(\frac{|\tau|}{\tau_0}\right)$$

- which means that the oscillations can be related to the source separation S and the distance to object L
- Know one - get the other!