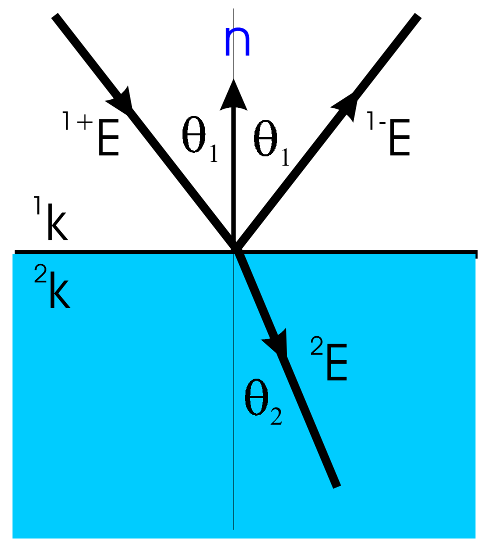
Lecture 11

Conducting Interfaces and Rough Surfaces

Boundary Conditions



The Story So Far...

- The incident wave (Lecture 9)
  - $^{1+}\mathbf{k} = (^{1+}k_x, ^{1+}k_y, ^{1+}k_z) = (^{1}k\sin\theta_1 , 0 , ^{1}k\cos\theta_1)$
- At the interface
  - Require spatial and temporal continuity
  - $^{1+}\mathbf{k} . \mathbf{r} - \omega t = ^{1-}\mathbf{k} . \mathbf{r} - \omega t = ^{2}\mathbf{k} . \mathbf{r} - \omega t$
  - no y components
  - no z component at interface (z=0)
  - $^{1+}k_x\, x = ^{1-}k_x\, x = ^{2}k_x\, x$
  - All the x components are equal
- We get =>
  - $\sin\theta_1 = \sin\theta_{1-}$ - Law of reflection
  - $n_1 \sin\theta_1 = n_2 \sin\theta_2$ - Snell's Law (n sinθ is conserved)

The Story So Far (II)...

- Boundary conditions for fields at the interface (Lecture 10)
- Tangential E and H fields are continuous
  - $(^{1+}\mathbf{E} + ^{1-}\mathbf{E}) \times \mathbf{n} = ^{2}\mathbf{E} \times \mathbf{n}$
  - $(^{1+}\mathbf{H} + ^{1-}\mathbf{H}) \times \mathbf{n} = ^{2}\mathbf{H} \times \mathbf{n}$
  - $(^{1+}\mathbf{k} \times ^{1+}\mathbf{E}) \times \mathbf{n} + (^{1-}\mathbf{k} \times ^{1-}\mathbf{E}) \times \mathbf{n} = (^{2}\mathbf{k} \times ^{2}\mathbf{E}) \times \mathbf{n}$
- We resolved components of **E** or **H** fields for $^{1-}\mathbf{k}$ and $^{2}\mathbf{k}$.
- But, we noticed that there was a critical angle $\sin\theta_c = n_2/n_1$
- where there was no spatially varying z component of $^{2}\mathbf{k}$.

Evanescent TE Waves

- What's $\cos\theta_2$?
- Snell's law: $\sin\theta_2=(n_1/n_2)\sin\theta_1 = \sin\theta_1/\sin\theta_c$
- $\cos^2\theta_2 = (1 - (n_1/n_2)^2\sin^2\theta_1)= (1 - \sin^2\theta_1/\sin^2\theta_c)$
  - $\cos\theta_2$ is imaginary
  - let $\beta^2 = {}^2k^2 (\sin^2\theta_1/\sin^2\theta_c - 1) = - {}^2k^2 \cos^2\theta_2$
- Transmitted wave propagates spatially as exp(i $\mathbf{{}^2k}$. $\mathbf{r}$)
- $\exp(i (x\, {}^2k \sin\theta_2 + z\, {}^2k \cos\theta_2)$
- $\exp(-\beta z) \exp(i (x\, {}^1k \sin\theta_1))$
- Wave decays on space scale $1/\beta$
- for glass n_2 =1.5 interface with air n_1 = 1
  - $\theta_c = \sin^{-1}(2/3) = 41.8^o$
  - at 45^o , 1/p about $\lambda_0/2$
- Evanescent wave is the wave that penetrates the second medium decaying as $1/\beta$

Evanescent TE Waves - Phase Changes

- Look at boundary conditions
  - ${}^{1+}E_0 + {}^{1-}E_0 = {}^2E_0$
  - ${}^{1+}E_0\, {}^{1+}k \cos\theta_1 - {}^{1-}E_0\, {}^{1-}k \cos\theta_1 = {}^2E_0\, {}^2k \cos\theta_2$
  - $\beta = i\, {}^2k \cos\theta_2$, $\alpha = {}^1k \cos\theta_1$
- Solve for ${}^{1-}E_0$,${}^2E_0$
  - ${}^2E_0 / {}^{1+}E_0 = 2\beta/(\alpha - i\beta)$
  - ${}^{1-}E_0 / {}^{1+}E_0 = (\beta + iq)/(\alpha - i\beta)$
  - There are phase changes in the reflected beam
- Look at R, T
  - T = 0 because there is no z component in transmitted wave
  - $R = |{}^{1-}E_0 / {}^{1+}E_0|^2 = |({}^{1-}E_0 / {}^{1+}E_0)({}^{1-}E_0 / {}^{1+}E_0)^*| = 1$
- Total internal reflection as all beam energy is reflected
- Occurs when $\theta_1 > \theta_c$

Reflection at a Conducting Interface

- Equations get complex - need approximations
- Normal Incidence - gets rid of angle effects
- $\mu_1 = \mu_2 = \mu_0$ (what about ferromagnetics?)
- Large conductivity σ (not bad for metals)

Reflection at a Conducting Interface

- Boundary Conditions
  - ${}^{1+}E_0 + {}^{1-}E_0 = {}^2E_0$
  - ${}^{1+}E_0({}^{1+}k /\mu_1) \cos\theta_1 - {}^{1-}E_0 ({}^{1-}k /\mu_1) \cos\theta_1 = {}^2E_0 ({}^2k / \mu_2) \cos\theta_2$
- Not yet assuming that $\mu_1 = \mu_2$
- Now ${}^2k$ is complex
  - ${}^2k^2 = \omega^2 (\mu\epsilon) = \omega^2 (\mu_2\epsilon_2) (1 + i\sigma_2/(\epsilon_2\omega))$
- Can get the angles from
  - $\sin\theta_2 = n_1 \sin\theta_1 / (n_2 + i\kappa_2)$
  - But it makes the solution messy
  - Assume normal incidence!

Reflection at a Conducting Interface

- Boundary Conditions
  - ${}^{1+}E_0 + {}^{1-}E_0 = {}^{2}E_0$
  - ${}^{1+}E_0\,({}^{1+}k/\mu_1) - {}^{1-}E_0\,({}^{1-}k/\mu_1) = {}^{2}E_0\,({}^{2}k/\mu_2)$
  - ${}^{2}k^2 = \omega^2\,(\mu_2\epsilon_2)\,(1 + i\sigma_2/(\epsilon_2\omega))$
- Still very complicated!!
- In metals at low frequencies contributions of bound electrons negligible compared with conducting electrons
  - So assume $\sqrt{\epsilon_2} = n + i\kappa = \sqrt{(i\sigma_2/\omega\,\epsilon_0)}$
  - $\sqrt{i} = (1 + i)/\sqrt{2}$
  - ${}^{2}k = (1 + i)\,\sqrt{(\sigma_2\,\mu_2\omega/2)}$
- Soluble (sort of!!)

Reflection at a Conducting Interface

- The solution is left as an exercise...
- We can solve for the reflected component $({}^{1-}E_0/{}^{1+}E_0)$
- $R = |({}^{1-}E_0/{}^{1+}E_0)({}^{1-}E_0/{}^{1+}E_0)^*| \approx 1 - 2\sqrt{(2\epsilon_1\omega/\sigma_2)}$
  - The higher the conductivity the higher is R
  - The lower the frequency the higher is R
- Good conductors have little or no z-component to the reflected beam
- $T = 2\sqrt{(2\epsilon_1\omega/\sigma_2)}$ energy is dissapated in metal as Joule heating
- Assumed that the conductivity is the DC value

A Simple Example

- We can extend our earlier analysis for a simple case of light incident on a metal in a vacuum. $n_1=1$, $n_2 = n + i\kappa$
- The Reflectance R becomes
- $R = |({}^{1-}E_0/{}^{1+}E_0)({}^{1-}E_0/{}^{1+}E_0)^*| = ((n-1)^2 + \kappa^2)/((n+1)^2 + \kappa^2)$
- $\kappa = 0$ => dielectric case back
- $\kappa >> n$ => Reflected wave → 1
- Sodium λ = 589.3nm, n=0.04, κ=2.4 T=0.1
- Bulk Tin λ = 589.3nm, n=1.5, κ=5.3 T=0.2
- Gallium λ = 589.3nm, n=3.7, κ=5.4 T=0.3

Reflection from Non-Flat Surfaces

- All surfaces can be considered to be superposition of sinusoidal surfaces (Fourier analysis)
- A surface height function h(x) can be considered as Fourier components (given some conditions)
- The general series is of the form
  - $z = h(x) = (1/2\pi)\int g(q)\exp(iqx)\,dq$
- If h(x) is periodic
  - $z = h(x) = \sum G(n)\exp(i\,n2\pi x/\Lambda)$ for $n = \{-\infty, ..., \infty\}$
- Consider one such surface for which $G(n)\neq 0$ for only 2 components

The Sinusoidal Surface

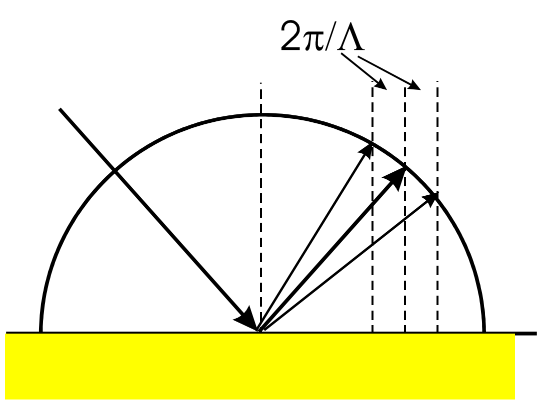
- Sinusoidal surface of perfect conductor (T = 0)
- Not strictly rigorous, but a limiting case
- Sum of incident and reflected fields = 0
  - ${}^{1+}E_0 \exp(i(k_x x+k_z z)) + {}^{1-}E_0(x,z) = 0$
    - on the surface $z = h(x) = h_0 \cos(2\pi x/\Lambda)$
    - $h(x) = h_0/2\ (\exp(2\pi i x/\Lambda) + \exp(-2\pi i x/\Lambda))$
- Solves at the boundary at z = h(x) to
  - ${}^{1-}E_0(x) = -\,{}^{1+}E_0 \exp(ik_x x)\exp(ik_z h_0 \cos(2\pi x/\Lambda))$

Reflection from Non-Flat Surfaces

- Look at simple solution for $k_z h_0 << 1$ ($\exp(x) = 1 + x$)
  - ${}^{1-}E_0(x) = -\,{}^{1+}E_0 \exp(ik_x x)(1 + ik_z h_0 \cos(2\pi x/\Lambda))$
  - Get the z component by conserving energy under total relfection
  - Rewrite cos as sum of two complex exponentials
  - Get three waves
    - k_x, $k_x + 2\pi/\Lambda$, $k_x - 2\pi/\Lambda$
    - Waves are at different angles
    - Diffraction grating
    - If we relax $k_z h_0 << 1$, get $k_x + 2m\pi/\Lambda$
- Rough surface can be simulated by summing sinusoids

Reflection from Non-Flat Surfaces

- Sinusoidal surface scatters at particular angles in steps of $2\pi/\Lambda$



"Real World" Reflection

- Examples of "typical" surface scattering by different types of surface (lobes represent polar diagrams of the scattered power)