Lecture 9

Transmission and Reflection

Reflection at a Boundary

- A boundary is defined as a place where something is discontinuous
- Half the work is sorting out what is continuous and what is discontinuous at the boundary
- At an optical boundary the refractive index n changes
- Leads to a wave returning (reflected) and a wave ongoing (transmitted)

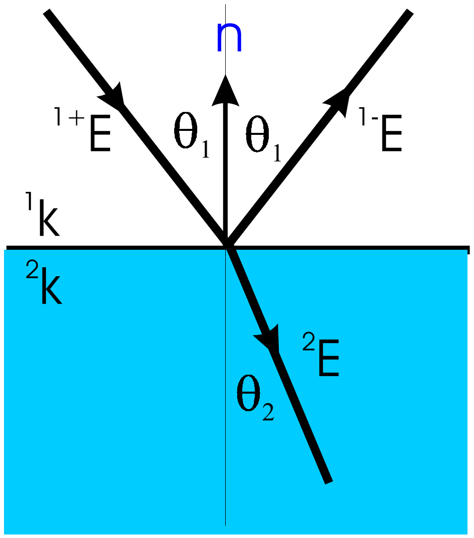
Reflection at a Boundary

- Continuous **across** the boundary
  - frequency
- Continuous **at** the boundary
  - spatial variation
  - For a dielectric
    - tangential E field
    - tangential H field

Specific Boundary

- Consider a boundary at z=0 with a plane wave incident on it
  - angle of incidence θ wrt to z-axis (normal)
  - electric vector in the x-z plane
  - No y variation
- We must have in the incident waves
  - ${}^{1}k_x = {}^{1}k\sin\theta_1$
  - ${}^{1}k_z = {}^{1}k\cos\theta_1$

Specific Boundary



At the Boundary

- At the interface
  - $\mathbf{k_1} \cdot \mathbf{r} - \omega t = \mathbf{k_{1-}} \cdot \mathbf{r} - \omega t = \mathbf{k_2} \cdot \mathbf{r} - \omega t$
  - no y components
  - no z component at interface (z=0)
  - ${}^{1+}k_x\, x = {}^{1-}k_x\, x = {}^{2}k_x\, x$
- For the reflected wave
  - $\sin\theta_1 = \sin\theta_{1-}$ - Law of reflection
  - $n_1 \sin\theta_1 = n_2 \sin\theta_2$ - Snell's Law (n sinθ is conserved)

Complex Refractive Index

- If n_2 is complex (conductor, etc) then life gets fun!!
  - $\sin\theta_2 = n_1 \sin\theta_1 / (n_2 + i\kappa_2)$
- Leads to the following expression for the propagation
- $\omega/c\, [\, x\, n_1 \sin\theta_1 + z\, p\, (n_2 \cos q - \kappa_2 \sin q)$
  $+ izp(\kappa_2 \cos q + n_2 \sin q)]$
- p, q are functions of n_1, n_2, κ_2, θ_1 but **not** x, z
- wave propagates spatially as $\exp(i\, \mathbf{k_2} \cdot \mathbf{r})$
- amplitude given by imaginary part
- phase given by real part

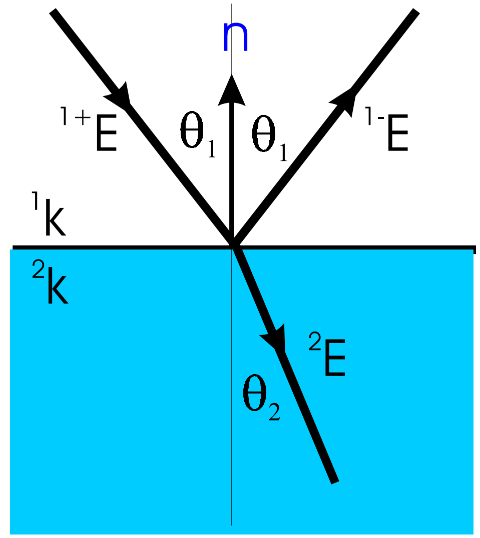
Inhomogeneous Waves

- Wave propagates as
- $\omega/c\, [\, x\, n_1 \sin\theta_1 + z\, p\, (n_2 \cos q - \kappa_2 \sin q)$
  $+ izp(\kappa_2 \cos q + n_2 \sin q)]$
- amplitude given by complex part of equation
  - surfaces of constant amplitude given by z=constant
- phase given by real part
  - surfaces of constant phase given by
    $x\, n_1 \sin\theta_1 + z\, p\, (n_2 \cos q - \kappa_2 \sin q) = \text{constant}$
- surfaces of constant phase not parallel to surfaces of constant amplitude
  - inhomogeneous waves

Boundary Conditions

- Boundary conditions at the interface
- Boundary conditions for dielectrics
  - Tangential E and H fields are continuous
  - $({}^{1+}\mathbf{E} + {}^{1-}\mathbf{E}) \times \mathbf{n} = {}^{2}\mathbf{E} \times \mathbf{n}$
  - $({}^{1+}\mathbf{H} + {}^{1-}\mathbf{H}) \times \mathbf{n} = {}^{2}\mathbf{H} \times \mathbf{n}$
  - $({}^{1+}\mathbf{k} \times {}^{1+}\mathbf{E}) \times \mathbf{n} + ({}^{1-}\mathbf{k} \times {}^{1-}\mathbf{E}) \times \mathbf{n} = ({}^{2}\mathbf{k} \times {}^{2}\mathbf{E}) \times \mathbf{n}$
- 1^{st} Case - Electric field parallel to interface
  - transverse electric (TE)
  - s-polarised
- 2^{nd} Case - Magnetic field parallel to interface
  - transverse magnetic field
  - p-polarised - (magnets have poles)

Boundary Conditions



TE Wave Reflection/Transmission

- Substitute in equations (assuming that $\mu_1 = \mu_2$)
  - ${}^{1+}E_0 + {}^{1-}E_0 = {}^{2}E_0$
  - ${}^{1+}E_0\, {}^{1+}k \cos\theta_1 - {}^{1-}E_0\, {}^{1-}k \cos\theta_1 = {}^{2}E_0\, {}^{2}k \cos\theta_2$
- Substitute for $k = 2\pi/\lambda_0\, n$
  - ${}^{1-}E_0 / {}^{1+}E_0 = [n_1\cos\theta_1 - n_2\cos\theta_2]/[n_1\cos\theta_1 + n_2\cos\theta_2]$
  - ${}^{2}E_0 / {}^{1+}E_0 = [2\, n_1\cos\theta_1]/[n_1\cos\theta_1 + n_2\cos\theta_2]$
- Substitute from Snell's Law
  - ${}^{1-}E_0 / {}^{1+}E_0 = \sin(\theta_2 - \theta_1)/\sin(\theta_2 + \theta_1)$
  - ${}^{2}E_0 / {}^{1+}E_0 = 2 \cos\theta_1 \sin\theta_2/\sin(\theta_2 + \theta_1)$
  - Fresnel Relations

At Normal Incidence

- ${}^{1-}E_0 / {}^{1+}E_0 = \sin(\theta_2 - \theta_1)/\sin(\theta_2 + \theta_1)$
- ${}^{2}E_0 / {}^{1+}E_0 = 2 \cos\theta_1 \sin\theta_2/\sin(\theta_2 + \theta_1)$
- At normal incidence - expressions are zero!!
  - or are they?
- as $\theta \to 0$
  - ${}^{1-}E_0 / {}^{1+}E_0 \to (\theta_2 - \theta_1)/(\theta_2 + \theta_1) \to (n_1 - n_2)/(n_2 + n_1)$
  - ${}^{2}E_0 / {}^{1+}E_0 \to 2\,\theta_2/(\theta_2 + \theta_1) \to 2n_1/(n_2 + n_1)$

Energy Transmission

- The energy reflection R, is given by...
  - resolve Poynting vector along normal
  - $- \mathbf{n}.\langle {}^{1-}\mathbf{S}\rangle / \mathbf{n}.\langle {}^{1+}\mathbf{S}\rangle$
    $= (\cos\theta_1\, n_1 |{}^{1-}E_0|^2)/(\cos\theta_1\, n_1 |{}^{1+}E_0|^2) = |{}^{1-}E_0|^2/|{}^{1+}E_0|^2$
    $= \sin^2(\theta_2 - \theta_1)/\sin^2(\theta_2 + \theta_1)$
- The energy transmission T, is given by...
  - resolve Poynting vector along normal
  - $\mathbf{n}.\langle {}^{2}\mathbf{S}\rangle / \mathbf{n}.\langle {}^{1+}\mathbf{S}\rangle$
    $= (\cos\theta_2\, n_2 |{}^{2}E_0|^2)/(\cos\theta_1\, n_1 |{}^{1+}E_0|^2)$
    $= \sin 2\theta_2 \sin 2\theta_1 / \sin^2(\theta_2 + \theta_1)$
- Notice that R + T = 1

At Normal Incidence

- $R = (n_2 - n_1)^2/(n_2 + n_1)^2$
- $T = 4n_1n_2/(n_2 + n_1)^2$
- R + T = 1

Summary for a TE wave

- For a dielectric interface there is no phase change (0,π)
- ${}^{1-}E_0 / {}^{1+}E_0 = \sin(\theta_2 - \theta_1)/\sin(\theta_2 + \theta_1)$
- ${}^{2}E_0 / {}^{1+}E_0 = 2\cos\theta_1 \sin\theta_2/\sin(\theta_2 + \theta_1)$
- At normal incidence
  - ${}^{1-}E_0 / {}^{1+}E_0 \rightarrow (\theta_2 - \theta_1)/(\theta_2 + \theta_1) \rightarrow (n_1 - n_2)/(n_2 + n_1)$
  - ${}^{2}E_0 / {}^{1+}E_0 \rightarrow 2\,\theta_2/(\theta_2 + \theta_1) \rightarrow 2n_1/(n_2 + n_1)$
- The reflected power ratio
  - $R = \sin^2(\theta_2 - \theta_1)/\sin^2(\theta_2 + \theta_1)$
- The trasmitted power ratio
  - $T = \sin 2\theta_2 \sin 2\theta_1 / \sin^2(\theta_2 + \theta_1)$
- At normal incidence
  - $R = (n_2 - n_1)^2/(n_2 + n_1)^2$, $T = 4n_1n_2/(n_2 + n_1)^2$

The Other Case (TM Waves)

- Maxwell’s Equations are (nearly) symmetrical in E, H if there are no free charges
- Any solution for E,H can be written for H,E if we interchange -μ, ϵ at the same time
- So if for the TE case we have (assuming that $\mu_1 = \mu_2$)
  - ${}^{1+}E_0 + {}^{1-}E_0 = {}^{2}E_0$
  - ${}^{1+}E_0\, {}^{1+}k \cos\theta_1 - {}^{1-}E_0\, {}^{1-}k \cos\theta_1 = {}^{2}E_0\, {}^{2}k \cos\theta_2$
- for the TM case it is...
  - ${}^{1+}k\, {}^{1+}E_0 + {}^{1-}k\, {}^{1-}E_0 = {}^{2}k\, {}^{2}E_0$
  - ${}^{1+}E_0 \cos\theta_1 - {}^{1-}E_0 \cos\theta_1 = {}^{2}E_0 \cos\theta_2$
- And the solution proceeds...

Summary for a TM wave

- For a dielectric interface there is no phase change (0,π)
  - $^{1-}E_0 / {}^{1+}E_0 = \tan(\theta_1 - \theta_2)/\tan(\theta_2 + \theta_1)$
  - $^{2}E_0 / {}^{1+}E_0 = 2 \cos\theta_1 \sin\theta_2/\sin(\theta_2 + \theta_1)/\cos(\theta_1 - \theta_2)$
- At normal incidence
  - $^{1-}E_0 / {}^{1+}E_0 \to (\theta_1 - \theta_2)/(\theta_2 + \theta_1) \to (n_2 - n_1)/(n_2 + n_1)$
  - $^{2}E_0 / {}^{1+}E_0 \to 2\ \theta_1\ \theta_2/(\theta_2 + \theta_1) \to 2n_1/(n_2 + n_1)$
- The reflected power ratio
  - $R = \tan^2(\theta_1 - \theta_2)/\tan^2(\theta_2 + \theta_1)$
- The trasmitted power ratio
  - $T = \sin 2\theta_2 \sin 2\theta_1 / \sin^2(\theta_2 + \theta_1)$
- At normal incidence
  - $R = (n_1 - n_2)^2/(n_2 + n_1)^2$, $T = 4n_1n_2/(n_2 + n_1)^2$

Critical Angle

- $n_1 \sin\theta_1 = n_2 \sin\theta_2$ - Snell's Law (n sinθ is conserved)
- If $(n_1/n_2)\sin\theta_1 > 1$ then θ_2 does not exist?
- There is no transmitted ray
  - reflection must be perfect!!

Brewster's Angle

- For a TM wave the reflected power ratio
  - $R = \tan^2(\theta_1 - \theta_2)/\tan^2(\theta_2 + \theta_1)$
- If $(\theta_2 + \theta_1) = \pi/2$ then $R = 0$
  - Perfect Transmission (TM only)
- For a TE wave the reflected power ratio
  - $R = \sin^2(\theta_2 - \theta_1)/\sin^2(\theta_2 + \theta_1)$
  - no minimum

Phase Changes

- For TE waves
  - $^{1-}E_0 / {}^{1+}E_0 = \sin(\theta_2 - \theta_1)/\sin(\theta_2 + \theta_1)$
  - $^{2}E_0 / {}^{1+}E_0 = 2 \cos\theta_1 \sin\theta_2/\sin(\theta_2 + \theta_1)$
  - Phase change on transmission is 0
  - Phase change on reflection is 0 if $\theta_2 > \theta_1$, π if $\theta_2 < \theta_1$
- For TM waves
  - $^{1-}E_0 / {}^{1+}E_0 = \tan(\theta_1 - \theta_2)/\tan(\theta_2 + \theta_1)$
  - $^{2}E_0 / {}^{1+}E_0 = 2 \cos\theta_1 \sin\theta_2/\sin(\theta_2 + \theta_1)/\cos(\theta_1 - \theta_2)$
  - Phase change on transmission is 0
  - Phase change on reflection
    - 0 if $(\theta_2 + \theta_1) < \pi/2$ (less than Brewster's)
    - π if $(\theta_2 + \theta_1) > \pi/2$ (greater than Brewster's)