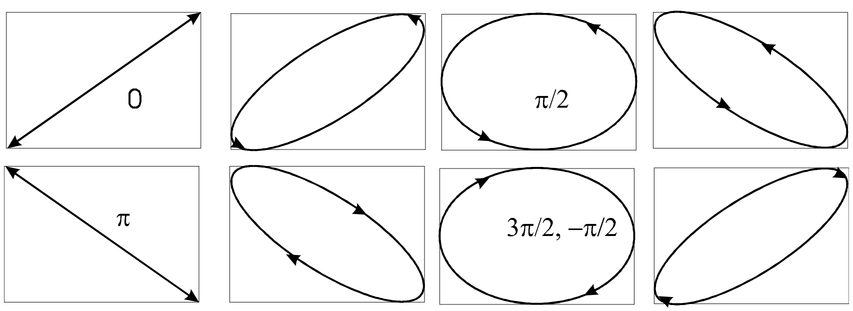
Lecture 7

Polarisation, Anisotropy and Jones Matrices

Some Common Polarisations

- 0, $m\pi$ - total electric vector is a straight line at an angle given by $\tan^{-1}(E_y/E_x)$ - linearly polarised
- $(2m+1)\pi/2$ and $E_y = E_x$ - circularly polarised
- Anything else - elliptically polarised light
- All described by the locus of the vector
- $E_x\cos\omega t\ \mathbf{x} + E_y\cos(\omega t+\delta)\ \mathbf{y}$
- Or in complex notation
  $E_x \exp i\omega t\ \mathbf{x} + E_y\exp i(\omega t+\delta)\ \mathbf{y} \rightarrow E_x\ \mathbf{x} + E_y\exp i\delta\ \mathbf{y}$

Some Common Polarisations



But You Can Only Measure Energy

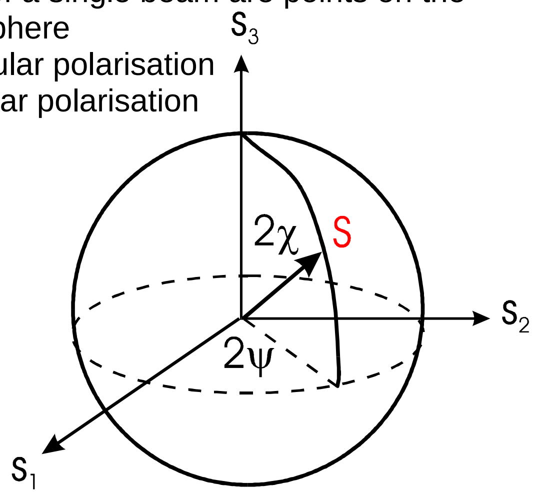
- OK so we must characterise things in terms of energy quantities

- $s_o = E_{ox}^2 + E_{oy}^2 = s_0$ Total energy
- $s_1 = E_{ox}^2 - E_{oy}^2 = s_0 \sin2\chi\cos2\psi$ Difference in energy
- $s_2 = 2\ E_{ox}\ E_{oy}\cos(-\delta) = s_0 \sin2\chi\sin2\psi$
- $s_3 = 2\ E_{ox}\ E_{oy}\sin(-\delta) = s_0 \cos2\chi$

- Stokes Parameters - after Lord Stokes.

Poincaré Sphere

- $s_0^2 = s_1^2 + s_2^2 + s_3^2$
- The angles 2χ and 2ψ form the latitude and longitude on the sphere
- All polarisations of a single beam are points on the surfaces of the sphere
- Eg poles are circular polarisation
- Eg equator is linear polarisation



The Real World

- See a sum of lots of little “wavelets” (except lasers!)
- Stokes parameters are energy-like quantities and can be added up for the sum of a set of “wavelets”
- $s_0 = \langle E_{ox}^2 \rangle + \langle E_{oy}^2 \rangle \qquad = s_0$
- $s_1 = \langle E_{ox}^2 \rangle - \langle E_{oy}^2 \rangle \qquad = s_0 \langle \sin 2\chi \cos 2\psi \rangle$
- $s_2 = 2 \langle E_{ox} E_{oy} \cos(-\delta) \rangle \qquad = s_0 \langle \sin 2\chi \sin 2\psi \rangle$
- $s_3 = 2 \langle E_{ox} E_{oy} \sin(-\delta) \rangle \qquad = s_0 \langle \cos 2\chi \rangle$

Natural Light

- “Natural” light does not follow the restrictions on frequency and phase as previous discussion
- “Natural” light considered as random superposition of “wavelets” of random phasing
- $s_0^2 \geq s_1^2 + s_2^2 + s_3^2$
- For light from a thermal source
- $s_0 <> 0, \; s_1 = s_2 = s_3 = 0$
- For white light from a linear polariser
- $s_0 = \pm s_1 <> 0, \; s_2 = s_3 = 0$

Anisotropy

- $\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$
- Without doing too much math. The permittivity and refractive index in the “right” co-ordinates can be written as a diagonal matrix (three values)
- If two diagonal elements are the same -> unixial
- If they’re all different -> biaxial
- That means that we can propagate E_x, E_y at different velocities and the phase relationship will depend upon the distance from the ref. point
- NOTE 1: The axes are controlled by the dielectric (crystal) axes - they are no longer arbitrary
- NOTE 2: Need to orient the crystal in the system to get all the axes right

Jones Matrices

- Convenient to write E_x, E_y as a column vector $\begin{pmatrix} E_x \\ E_y \end{pmatrix}$
- Here are some Jones vectors

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{Linear} \qquad \sqrt{1/2}\begin{pmatrix} 1 \\ \pm i \end{pmatrix} \quad \text{Circular}$$

- Jones Vectors can be added BUT only if everything refers to the same (exactly the same) frequency (defined phase relationship)
- An optical element can be configured as a 2x2 matrix which multiplies the Jones vector to give a new vector
- Optical elements can be stacked to find the effect of a system on polarisation

Some Jones Matrices

- Linear Polarisers

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$$

- Phase Changer

$$\begin{pmatrix} 1 & 0 \\ 0 & \exp(i\delta) \end{pmatrix}$$

Produce Polarised Light

- Start with anything
- Linearly polarise it - generates an axis set
- Add a phase retarder where $\delta = \pi/2$
- Called a “quarter wave plate because the relative retardation is 1/4 of a wavelength
- Got circularly polarised light!

Optical Activity

- Substances can rotate the plane of polarisation
- Amount of rotation per unit length = “specific rotary power” eg 3.7^{o}/mm
- Can be “explained” as a different propagation speed
- (refractive index) for RH and LH circular polarisation
- How does this happen?
- Effect of a magnetic field on propagation

Optical Activity

- Notice that this is a static external field, not the field from the wave itself
- Using our SHM model of the dielectric
- m d^2/dt^2 (**r**) + K**r** = -e**E** - e d**r**/dt x (μ$\mathbf{H}_0$)
- But the polarisation **P** = ne**r**
- So can solve the above for a wave solution
- $(-m\omega^2 + K)\,\mathbf{P} = Ne^2\mathbf{E} + i\omega\mu\,\mathbf{P} \times \mathbf{H}_0$
- Which can be written in the form of a tensor
- $$\begin{matrix} \chi_{xx} & i\chi_{xy} & 0 \\ -i\chi_{xy} & \chi_{xx} & 0 \\ 0 & 0 & \chi_{zz} \end{matrix}$$
- Where all the χ are functions of the field H_0
- We can solve for the dispersion relation for the two polarisations...

Eigenstates, etc.

- $-k^2 E_x + \omega^2/c^2 E_x = -\omega^2/c^2 (\chi_{xx} E_x + i\chi_{xy} E_y)$
- $-k^2 E_y + \omega^2/c^2 E_y = -\omega^2/c^2 (-i\chi_{xy} E_x + \chi_{xx} E_y)$
- Which has solutions for
- $E_x = \pm i E_y$
- And $k = (\omega/c)\sqrt{(1+\chi_{xx} \pm \chi_{xy})}$
- Which leads to
- $n_\ell = \sqrt{(1+\chi_{xx} + \chi_{xy})}$
- $n_r = \sqrt{(1+\chi_{xx} - \chi_{xy})}$

Optical Activity - Natural

- Birefringence - multiple refractive indices (linear)
- Optical activity - multiple refractive indices (circular)

Optical Activity - Induced

- Faraday rotation
  - Apply a magnetic field
  - Becomes optically active
- Voigt Effect
  - Apply a magnetic field
  - Becomes birefringent
- Pockels Effect - material has no centre of inversion (crystal)
  - Apply an electric field
  - Becomes (changes) birefringence
- Kerr Effect - material does have a centre of inversion (isotropic)
  - Apply an electric field
  - Becomes birefringent
- Above are used routinely to manipulate (switch) laser beams - high frequency operation

Typical Kerr Cell

- Linearly polarise light going in
- When Kerr cell off (no field) nothing happens
- Kerr cell axis at 45^o to x,y
- Apply field to apply π relative phase shift between E_x, E_y
- Then polarisation is still linear but rotates 180^o
- Use a second polariser at 90^o to first
- Blocks off, Passes on....