# Lecture 5

## Plasmas, Metals and Dielectrics

# Equation of Motion

- $m\, \partial^2 \mathbf{r}/\partial t^2 + m\gamma\, \partial \mathbf{r}/\partial t + K\mathbf{r} = -e\mathbf{E}$
- If material is a metal K = 0
- If material is a plasma K,γ = 0
- Assume sinusoidal time dependence, frequency ω
- $- \omega^2 m\, \mathbf{r} - i\omega\, m\gamma\, \mathbf{r} + K\mathbf{r} = -e\mathbf{E}$
- Note that **r** is still proportional to **E**
- $\mathbf{r} = e\mathbf{E} / (\omega^2 m - K + i\omega\, m\gamma)$

# Velocity and Conductivity

- Now Let's look at the velocity **v** (diff. **r** wrt t)
- $\mathbf{v} = -e\mathbf{E} / (m\gamma + i(\omega m - K/\omega))$
- Now classically the current density **J** is related to the electric field **E**
- $\mathbf{J} = \sigma\mathbf{E} = -Ne\mathbf{v}$
- N is carrier density (Carriers per unit volume)
- $\sigma = Ne^2 (\omega/m) / (\gamma\omega - i(\omega^2 - K/m))$
- Let $K/m = \omega_0^2$, let $Ne^2/(\epsilon_0 m) = \omega_p^2$.
- $\sigma = \epsilon_0 \omega_p^2 \omega / (\gamma\omega + i(\omega_0^2 - \omega^2))$
- IF $\omega_0 = 0$, which implies K = 0 (metal, plasma),
- THEN ω→0 (DC) value is $\sigma(\omega \to 0) = \epsilon_0 \omega_p^2 / \gamma$
- Otherwise $\sigma(\omega \to 0) = 0$
- Enables me to "evaluate" $\omega_p^2 / \gamma$

# Connection to MEs

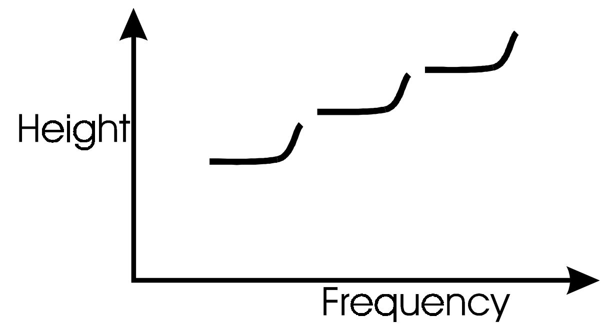
- We redefined $\epsilon + i\sigma/\omega$
- We said that nothing was happening except the electron motion
- $\epsilon = \epsilon_0 + \epsilon_0 \omega_p^2 / ((\omega_0^2 - \omega^2) - i\gamma\omega)$
- $\epsilon/\epsilon_0 = 1 + \omega_p^2 / ((\omega_0^2 - \omega^2) - i\gamma\omega) = n^2$
- So we can at least notionally compute the refractive index on this basis for three cases:
- Insulators, metals and plasmas

## Plasmas

- Gases in which electrons have separated from the parent atoms/molecules
- Meet them in laser fusion, upper atmosphere
- Ions stay “fixed”, electrons move (like cars and bikes in Toronto)
- $\omega_0 = \gamma = 0$, no restoring force, no damping
- $n^2 = 1 - \omega_p^2 / \omega^2$
- If $\omega < \omega_p$ , $n$ is imaginary, no propagation
- If $\omega > \omega_p$ , $n$ is real propagation with no attenuation
- Below plasma frequency $\omega_p$, waves are reflected at plasma boundary - above go through
- $\omega_p^2 = Ne^2/(\epsilon_0 m)$
- Plasma frequency is a measure of election density, N

## Ionospheric Sounding

- Point transmitter up - transmit pulse
- Measure time to get echo vs frequency
- Frequency of disappearance is plasma frequency
- Below plasma frequency - reflectance
- Above plasma frequency - transmitted
- Expect to get echo up to a certain frequency - then silence
- Can get a measure of electron density vs height




## Metals - Plasmas with Drag

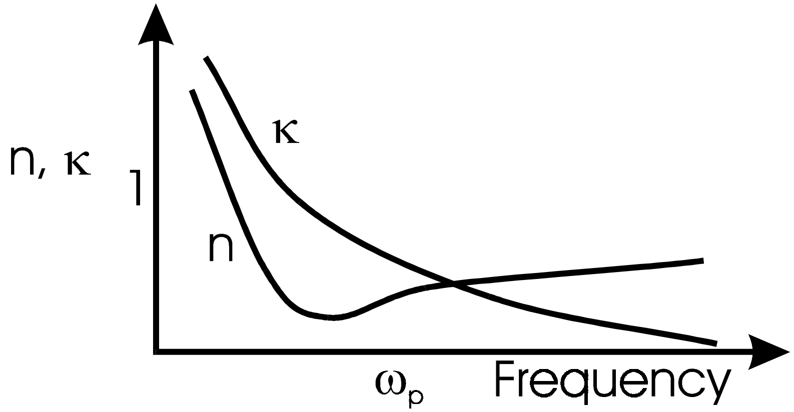
- In a metal K= 0 but $\gamma$ # 0 - there is a drag
- $n^2 = \epsilon/\epsilon_0 = 1 - \omega_p^2 / ( \omega^2 + i \gamma\omega )$
- $n^2 - \kappa^2 = 1 - \omega_p^2 / ( \omega^2 + \gamma^2 )$
- $2n\kappa = \omega_p^2 (\gamma/\omega)/ ( \omega^2 + \gamma^2)$
- At low frequencies
- Solving the above gives $\kappa = \surd(\omega_p^2 / ( 2\omega\gamma)) = n_i$
- SO WHAT!!!
- Remember this...
- $\exp(ikz) = \exp(i\, n_r (2\pi/\lambda) z) \exp(- n_i (2\pi/\lambda) z)$
- Now $2\pi/\lambda = \omega/c$
- So wave decays spatially as $\beta = c /(\kappa\omega)$
- $\beta = \surd( 2c^2\gamma / \omega_p^2 \omega)$
- $\sigma(\omega \to 0) = \epsilon_0 \omega_p^2 / \gamma$
- $\beta = \surd( 2c^2 \epsilon_0 / \sigma_{DC} )$
- $\beta = \surd(2.5 \times 10^6 / (f\pi^2\sigma_{DC} ))$

## Current is Only Skin Deep

- Since all oscillating current is controlled by MEs
- All current in conductors must follow this rule
- Spatial extent of current flow from surface is given by $\beta = \surd(2.5 \times 10^6 / (f\pi^2\sigma_{DC} ))$
- For silver $\sigma_{DC}$ of order 7 . $10^7$ ohm$^{-1}$ m$^{-1}$
- So skin depth at 60Hz is about 8mm!!
- Skin depth at 60MHz is about 8µm - most of the wire isn’t doing anything!!!
- For copper $\sigma_{DC}$ of order 5.8 x $10^7$ ohm$^{-1}$ m$^{-1}$
- So skin depth at 60Hz is about 8.5mm
- Power station bus bars are laminated
  - otherwise it’s a waste of copper
- High frequency wires can be tubes!!
- Or use multiple strands of insulated wire (Litz wire)
- Silver plate your copper for HF use?
- Hi-Fi addicts beware

## Metals at Higher Frequencies

- $n^2 = \epsilon/\epsilon_0 = 1 - \omega_p^2 / (\omega^2 + i\gamma\omega)$
- $n^2 - \kappa^2 = 1 - \omega_p^2 / (\omega^2 + \gamma^2)$
- $2n\kappa = \omega_p^2 (\gamma/\omega) / (\omega^2 + \gamma^2)$
- Solve for n and κ

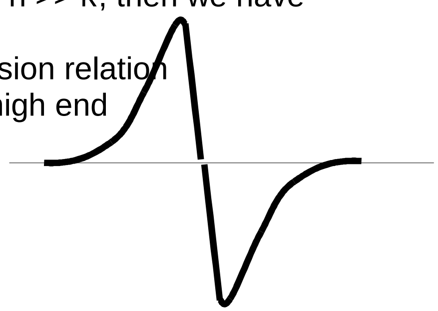



## Some Numerics

- $Ne^2/(\epsilon_0 m) = \omega_p^2$.
- $\sigma(\omega \to 0) = \epsilon_0 \omega_p^2 / \gamma$
- For silver N is about $10^{28}$ $m^{-3}$
- And $\sigma_{DC}$ is about 7 . $10^7$ $ohm^{-1}$ $m^{-1}$
- SO $f_p$ is about 9 x $10^{14}$ Hz or 333nm (near UV)
- AND γ is about 2 x $10^{12}$ $sec^{-1}$
- OR the damping time is about 0.5pS

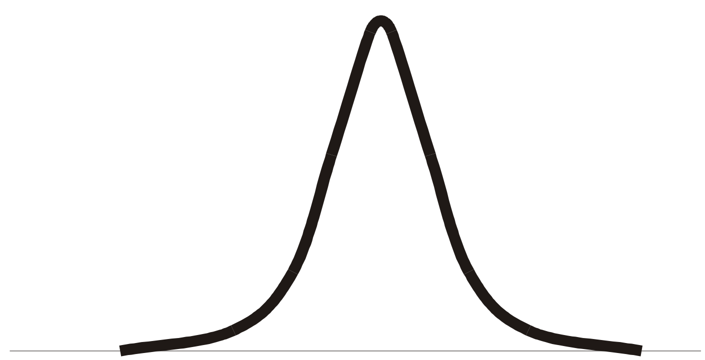
## Insulators - Virtual Conductors

- $\epsilon/\epsilon_0 = 1 + \omega_p^2 / ((\omega_0^2 - \omega^2) - i\gamma\omega) = n^2$
- Now we can't eliminate anything!!
- $n^2 - \kappa^2 = 1 + \omega_p^2 (\omega_0^2 - \omega^2) / ((\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2)$
- $2n\kappa = \omega_p^2 \gamma \omega / ((\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2)$
- These exhibit a resonance at $\omega_0$
- Solve around $\omega_0$ , then $\omega_0^2 - \omega^2 = 2\omega_{(0)} (\omega_0 - \omega)$
- $n^2 - \kappa^2 = 1 + (\omega_p^2/2\omega_0) (\omega_0 - \omega) / ((\omega_0 - \omega)^2 + \gamma^2/4)$
- $2n\kappa = (\omega_p^2/2\omega_0) (\gamma/2) / ((\omega_0 - \omega)^2 + \gamma^2/4)$
- If the scaling is such that n >> κ, then we have equations for n, κ
- First equation is a dispersion relation
- In actual fact low end > high end
- $n^2 - \kappa^2 = 0$ at $\omega_0$



## Lorentz Functions

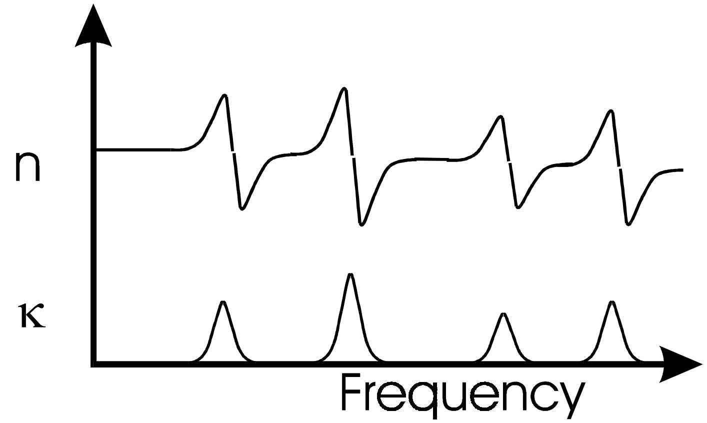
- $2n\kappa = (\omega_p^2/2\omega_0) . (\gamma/2) / ((\omega_0 - \omega)^2 + \gamma^2/4)$
- This is the Lorentz function - $1 / (x^2 + a^2)$
- $2n\kappa = (\omega_p^2/2\omega_0) / (\gamma/2)$ at $\omega_0$
- Width of function is γ/2
- Far from resonance
- $2n\kappa = (\omega_p^2/2\omega_0) . (\gamma/2) / (\omega_0 - \omega)^2$
- Going as $\omega^{-2}$



## Multiple Resonances

- In "real" dielectrics there are a number of resonances
- So we use principle of superposition to "add them up"
- Gives a series of dispersions in the real part of the refractive index with an overall downward trend with increasing frequency
- And a series of absorption "resonances"
- BUT locally, between resonances, n shows an increasing trend with frequency

## Dielectric Optical Properties




- Lorenz formula predicts that if there are resonances both above and below region of interest then there will be a gradual increase of refractive index with frequency or a gradual decrease of refractive index with wavelength

## Optical Properties of Some Glasses