Midterm #1 Solutions

PHY 238Y

Q1 (a) $\pi = CRT = 150 \frac{moles}{m^3} \times 8.31 \frac{J}{moles \cdot K} \times 300K = 3.74 \times 10^5 Pa$

$\pi \approx 3.74\, atm$

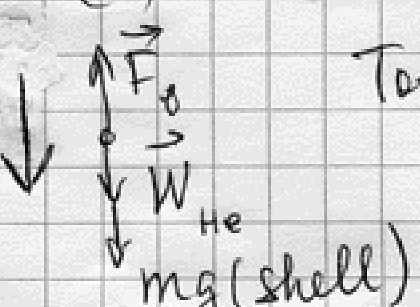
(b) $\pi = \rho g h \Rightarrow$ such solution can support a column of liquid of $h = \frac{\pi}{g\rho} = 38\, m$

* the density of the solution is approximated by that of water.

Q2 (a) the buoyant force on the object is equal to the weight of the displaced liquid: $V_b = \frac{4}{3}\pi(1.5m)^3$

$F_b = g \rho_a V_b = 165N$

(b) Balance of forces:



Total weight (shell + helium)

$\vec{W} = m\vec{g} + m_{He}\vec{g}$

$mg + m_{He}g - F_b = 0 \Rightarrow$

$m_{He} = \frac{F_b - mg}{g} = \frac{165N}{9.81 m/s^2} - 3kg \approx 13.8\, kg$

(c) Number of moles: $n = \frac{m_{He}}{m_{at}(He)} = \frac{13.8 \times 10^3 g}{4\, g/mole} = 3450$ moles

(d) Ideal gas law: $pV = nRT \Rightarrow p = \frac{nRT}{V} = 6.1 \times 10^5 Pa$

Q3 (a) an ideal fluid ⇒ use Bernoulli's principle; for a constant cross-section and elevation: no drop of pressure: $P_{left} = P_{right} = P_{atm} = 1.03\times10^5\,Pa$.

(b) a viscous flow:

If a flow is laminar ⇒ apply Poiseuille's law

$$Q = \frac{\Delta P \pi R^4}{8\eta l} \Rightarrow \Delta P = \frac{8\eta l Q}{\pi R^4}; \quad P_{left} = P_{atm} + \Delta P;$$

If a flow is turbuletn ⇒ cannot predict P_{left}.

For $Q = 9\times10^{-3}\,m/s^3$

$$\bar{v} = \frac{Q}{\pi R^2} \approx 70\,m/s$$

Reynolds number: $N_R = \frac{2R\rho\bar{v}}{\eta} \simeq 8.9\times10^5$ - turbulent -

cannot predict P_{left}

Q4 $R = 5\times10^{-3}$, $v_R = 0.15\,m/s$ - artery; ideal fluid.
$r = 5\times10^{-6}$, $v_r = 5\times10^{-4}\,m/s$ - capillaries; $Q = A\bar{v} =$ const (continuity)

$$\pi R^2 v_R = n\pi r^2 v_r$$

$$n = \frac{v_R}{v_r}\left(\frac{R}{r}\right)^2 = 3\times10^8$$

(a) artery : $N_R = 760 < 2000$ (laminar)
(b) capillaries: $N_R \ll 1$ - laminar too)

Q5 (a) The bubble has two surfaces.

$\Delta P = \frac{2\cdot 2\gamma}{r} \Rightarrow \gamma = \frac{\Delta P\, r}{4} = 2.25\times10^{-2}\ N/m$

(b) the droplet has one surface

The difference of pressures $\Delta P = \frac{2\gamma}{r} = 7280\ N/m^2$

If $\Delta P >$ P_{vapor}, the droplet cannot exist

$\Delta P = 7280\ N/m^2 > 2330\ Pa \Rightarrow$

the droplet will evaporate.