PHY 238 Y

QUESTIONS

Test # 3 Solutions

Question #1

(i) An ion source in a mass spectrometer produces deuterons. (A deuteron is a particle that has approximately twice the mass of a proton, but the same charge). Each deuteron is accelerated from rest through a potential difference of $2\text{x}10^3$ V, after which it enters a 0.60-T magnetic field. Find the radius of its circular path. Proton mass is $1.67\text{x}10^{-27}$ kg, elementary charge is $1.60\text{x}10^{-19}$ C.

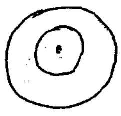
$$\frac{mv^2}{2} = eV \qquad m = 2m_p$$

$$\frac{mv^2}{R} = e\vec{v}\times\vec{B} = evB \qquad R = \frac{mv}{eB} = \sqrt{\frac{2Vm}{eB^2}}$$

$$R = \sqrt{\frac{4Vm_p}{eB^2}} = \sqrt{\frac{4\times 2\times10^3\,V\times 1.67\times10^{-27}\,kg}{1.6\times10^{-19}C\times 0.36T^2}} = 1.52\times10^{-2}\,m$$

(ii) Two circular coils are concentric and lie in the same plane. The inner coil contains 120 turns of wire, has a radius of 0.012 m, and carries a current of 6.0 A. The outer coil contains 150 turns and has a radius of 0.017 m. What must be the amplitude and direction (relative to the current in the inner coil) of the current in an outer coil, such that the net magnetic field at the common center of the two coils is zero?

- concentric coils (<u>not</u> solenoids!)



at center of circular arc:

$$|\vec{B}| = \mu_0 \frac{i\phi}{4\pi R} \; ; \quad \text{for a complete circle } \phi = 2\pi$$

$$|B| = \frac{\mu_0 2\pi i}{4\pi R} = \frac{\mu_0 i}{2R}$$

for a coil of N turns: $B = \frac{N\mu_0 i}{2R}$

For two concentric circles at their centre

$$\vec{B} = \vec{B_i} + \vec{B_o} = |B_i| \; |B_o| = \frac{\mu_0}{2}\left(\frac{i_i N_i}{R_i} - \frac{i_o N_o}{R_o}\right) = 0$$

$$i_o = \frac{i_i N_i R_o}{R_i N_o} = \frac{6A\times 120\times 0.017\,m}{0.012m\times 150} = 6.8A$$

- in opposite direction with respect to the current in the inner coil.

Question #2

(i) A circular coil of one turn is made from a wire of length 7.00×10^{-2} m. There is a current of 4.3 A in the wire. In the presence of a 2.50-T magnetic field, what is the largest torque that this loop can experience?

$$\ell = 2\pi r$$

$$A = \pi r^2 = \frac{\pi \ell^2}{4\pi^2} = \frac{\ell^2}{4\pi}$$

$$\mu = NiA$$

$$\vec{\tau} = \vec{\mu} \times \vec{B} = \mu B \sin\theta \; ; \quad \text{max value when } \theta = 90$$

$$|\tau|_{max} = NiAB = Ni\frac{\ell^2}{4\pi}B = \frac{1 \times 4.3A \times 2.5T \times 49 \times 10^{-4} m^2}{4 \times 3.14} =$$

$$= 4.2 \times 10^{-3} N \cdot m$$

(ii) A square loop of wire is held in a uniform magnetic field of 0.24 T directed perpendicularly to the plane of the loop. The length of the each side of the square is decreasing at a constant rate of 5 cm/s. What emf is induced in the loop when the length of each side is 12 cm?

$$\mathcal{E} = -N\frac{d\Phi}{dt} = -NB\frac{dA}{dt} = -NB\frac{d(\ell(t))^2}{dt}$$

$$\mathcal{E} = -NB\,2\,\ell(t)\frac{d\ell(t)}{dt} = \qquad \frac{d\ell}{dt} = -0.05 \, m/s$$

$$= +0.24T \times 2 \times 0.12m \times 0.05\, m/s$$

$$= 2.9 \times 10^{-3} V$$

Question #3

(i) The elements in a series RCL circuit are a 106-Ω resistor, a 3.30-μF capacitor, and a 0.0310-H inductor. The frequency is 609 Hz. What is

(a) The impedance of the circuit

$f = 609\ Hz$

$\omega = 2\pi f$ $\quad X_L = 2\pi f L = 118.56\ \Omega \approx 118.6\ \Omega$

$X_C = \frac{1}{2\pi f C} = 79.3\ \Omega$

$Z = \sqrt{(X_C - X_L)^2 + R^2} = 113\ \Omega$

and

(b) the phase angle between the current and the voltage of the generator.

$\tan\phi = \frac{X_L - X_C}{R} = 0.37 \Rightarrow \phi \approx 20°$

Question 3 (continued)

(ii) A series LCR circuit has a resonant frequency of 6.00 kHz. When it is driven at 8.00 kHz, it has an impedance of 1 kΩ and a phase constant of 45°. What are the values of

$f_r = 6\times10^3\,Hz$
$f_d = 8\times10^3\,Hz$

(a) R $\quad \frac{X_L - X_C}{R} = \tan 45° = 1 \Rightarrow X_L - X_C = R$

$Z^2 = (X_L - X_C)^2 + R^2 \Rightarrow Z^2 = 2R^2 \Rightarrow R = \frac{Z}{\sqrt{2}} = 707\,\Omega$

or

$\frac{R}{Z} = \cos\phi = \frac{1}{\sqrt{2}} \qquad R = 707\,\Omega$

(b) L $\quad X_L - X_C = R \Rightarrow 2\pi f_d L - \frac{1}{2\pi f_d C} = R$

divide both sides of the equation by L

$2\pi f_d - \frac{1}{2\pi f_d LC} = \frac{R}{L} \qquad LC = \frac{1}{4\pi^2 f_r^2}\,; \quad \frac{1}{LC} = 4\pi^2 f_r^2$

$2\pi f_d - \frac{\cancel{4\pi^2} f_r^2}{\cancel{2\pi f_d}} = R/L \Rightarrow \frac{2\pi(f_d^2 - f_r^2)}{f_d} = R/L$

$L = \frac{R f_d}{2\pi(f_d^2 - f_r^2)} = \frac{707\,\Omega \times 8\times10^3\,Hz}{6.28\times(64\times10^6 - 36\times10^6)} = 32.16\times10^{-3}\,H$

(c) C

for this circuit?

$LC = \frac{1}{4\pi^2 f_r^2}$

$C = \frac{1}{4\pi^2 f_r^2 L} = \frac{1 \times 10^3}{4\times3.14^2\times36\times10^6\times32.16} = 2.19\times10^{-5}\,F$