Midterm # 3 Solutions (PHY 238Y)

Electricity and Magnetism

QUESTIONS

Question #1

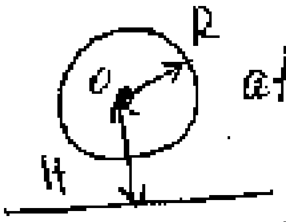
(i) A charged particle with a charge-to-mass ratio of $q/m = 5.7 \times 10^8$ C/kg travels on a circular path that is perpendicular to a magnetic field whose magnitude is 0.72T. How much time does it take for the particle to complete one revolution?

$\vec{F}_B = q\vec{v}\times\vec{B}$, $\vec{v} \perp \vec{B}$ for a circular orbit [8]

$\frac{mv^2}{r} = qvB \Rightarrow \frac{r}{v} = \frac{m}{qB}$; $\ell = 2\pi r = vT \Rightarrow$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{B(\frac{q}{m})} = \frac{2\times 3.14}{0.727T \times 5.7\times 10^8 \text{ C/kg}} = 1.5\times 10^{-8}\text{ s}$$

(ii) A circular loop of wire of a radius R and a long, straight wire carry currents I_1 and I_2 where $I_2 = 6.6I_1$. The loop and the straight wire lie in the same plane. Magnetic field in the centre of the loop is zero. Find the distance H between the straight wire and the center of the circular loop. Express your answer in terms of R.



at O: $\vec{B}_1 + \vec{B}_2 = 0$ $|\vec{B}_1| = |\vec{B}_2|$

magnetic field due to a straight wire: $B_2 = \frac{\mu_0 i_2}{2\pi H}$
(at the center of the circle)

magnetic field due to a circular current $B_1 = \frac{\mu_0 i_1 \phi}{4\pi R} = \frac{\mu_0 i}{2R}$
($\phi = 2\pi$)

$\frac{\mu_0 i_2}{2\pi H} = \frac{\mu_0 i_1}{2R} \Rightarrow$ solve for H in terms of R

$$H = \frac{i_1}{i_2}\frac{R}{\pi} = \frac{6.6R}{\pi} = 2.1R$$

Question #2

(i) A 0.5 – m length of wire is formed into a single-turn, square loop in which there is a current of 12 A. The loop is placed in a magnetic field of 0.12 T.

(a) What is the maximum torque that the loop can experience?

(b) At which orientation of the loop with respect to the direction of the magnetic field does it happen?

$\vec{\tau} = \vec{\mu} \times \vec{B}$; $|\vec{\mu}| = N \cdot i A$, $N = 1$, $L = 0.5$, $4\ell = L$

$|\vec{\tau}| = \frac{i L^2}{16} B = \frac{0.5^2 m^2 \times 12 A \times 0.12 T}{16} = 0.0225\ N \cdot m$

$|\vec{\tau}| = \mu B \sin\theta$ – maximum when $\theta = 90^\circ \Rightarrow \vec{\mu}$ is perpendicular to B (or normal to the loop is perpendicular to $\vec{B}$, or the plane of the loop is parallel to B)

(ii) A 75-turn conducting coil has an area of 8.5×10^{-3} m^2 and the normal to the coil is parallel to the magnetic field **B**. The coil has a resistance of 14 Ω. At what rate (in T/s) must the magnitude of **B** change for an induced current of 7.0 mA to exist in the coil?

Faraday's law:

$\mathcal{E} = -N\frac{d\Phi}{dt} = -NA\frac{dB}{dt}$

$\mathcal{E} = IR$

$\Rightarrow \left|\frac{dB}{dt}\right| = \frac{IR}{NA} = \frac{14\Omega \times 7 \times 10^{-3} A}{75 \times 8.5 \times 10^{-3} m^2} = 0.15 T/s$

Question #3

(i) A series RCL circuit contains a 47.0- Ω resistor, a 2.0-μF capacitor, and a 4-μH inductor. When the frequency is 2550 Hz, what is the power factor of the circuit?

$\omega = 2\pi f = 1.6\times10^4\ Hz$

$X_L = \omega L = 64\ \Omega$

$X_C = 1/(\omega C) = 31.2\ \Omega$

$\tan\varphi = \frac{X_L - X_C}{R} = 0.69$; $\varphi = \tan^{-1}\varphi = 35^\circ$

the power factor: $\cos\varphi = \cos 35^\circ \approx 0.82$

(ii) A series RLC circuit has a resonant frequency of 1500 Hz. When operating at a frequency other than 1500 Hz, the circuit has a capacitive reactance of 5.0 Ω and inductive reactance of 30.0 Ω. What are the values

(a) of *L* $f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow LC = \frac{1}{4\pi^2 f_0^2}$ (res.)

(b) of *C*

At $f \neq f_0$

$X_C = \frac{1}{2\pi f C}$ (1)

$X_L = 2\pi f L$ (2)

multiply (1)×(2): $X_C X_L = \frac{L}{C}$

$\frac{L}{C}\ LC = \frac{1}{4\pi^2 f_0^2} X_C X_L \Rightarrow$

$L = \frac{1}{2\pi f_0}\sqrt{X_L X_C} = 1.3\times10^{-3}\ H = 1.3\ mH$

Finally, $C = \frac{L}{X_L X_C} = 8.6\times10^{-6}\ F = 8.6\ \mu F$