Error Analysis

"To err is human; to describe the error properly is sublime."
- Cliff Swartz (1999)

Today

- A discussion about a laboratory topic: error analysis
- Your learning of this:

1. The assignment
2. Using error analysis in an experiment
3. This talk
4. A test (administered via computer)

Presenters: a "Tag Team"

Coming Next Week…

- We will begin the ***Waves Quarter*** on Oscillations, Sound and Light.
- For Monday, please read Sections 14.1 through 14.3 of Knight.
- There is a Pre-Class Quiz (Waves #1) on Chapter 14 due Monday morning on www.masteringphysics.com.

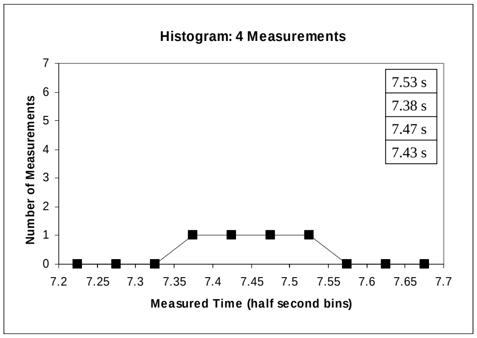
Two Kinds of Statements

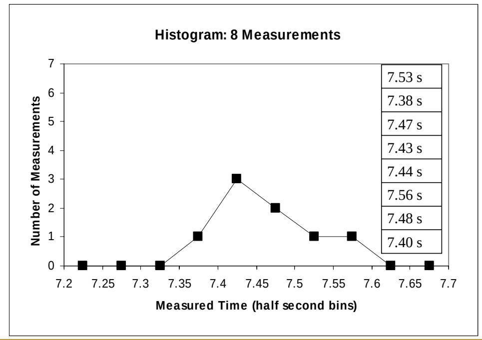
1. Exact
   - 2 + 3 = 5 (math)
   - $K = ½ mv^2$ (definition)
2. Approximate
   - $F_{spring} = -kx$ (*any* physical law)
   - $g = 9.80\ m/s^2$ (*all* numerical measures of the universe)

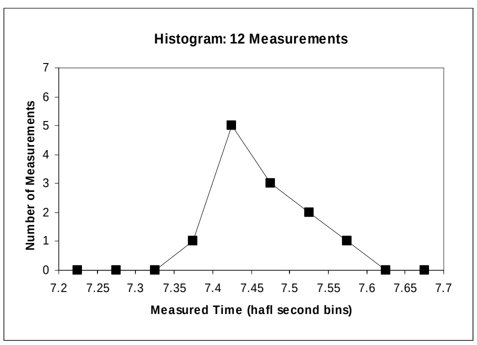
Today: approximate statements

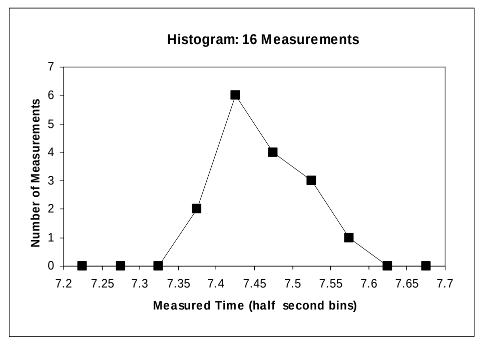
The t_5 data

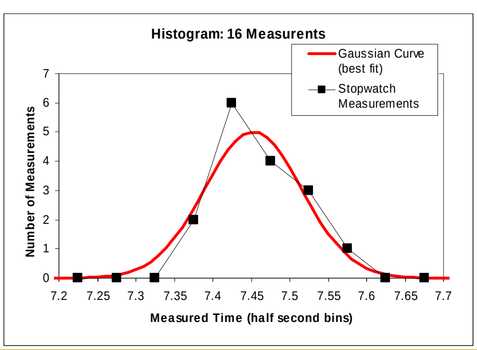
7.53 s
7.38 s
7.47 s
7.43 s







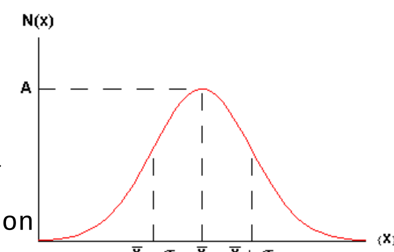




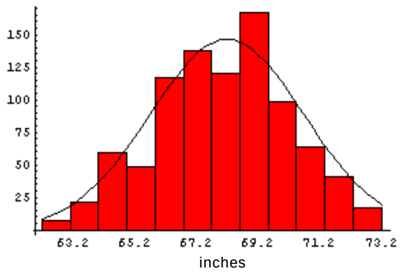
The Gaussian

$$N(x) = A\,e^{\frac{-(x-\bar{x})^2}{2\sigma^2}}$$

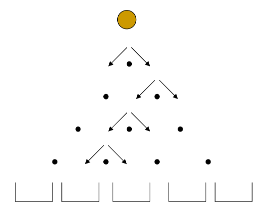
68% of data between the dotted lines on the graph.



Heights of some People (London, 1886)



Random Walk

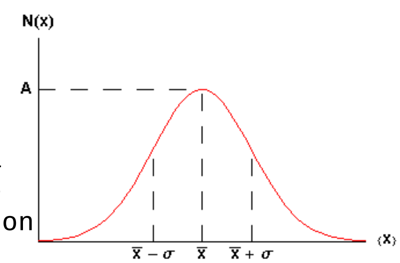


Where does an object end up, if it takes N steps randomly left or right?

The final distribution is described by a Gaussian function!

The Gaussian

$$N(x) = A\,e^{\frac{-(x-\bar{x})^2}{2\sigma^2}}$$



68% of data between the dotted lines on the graph.

The t_5 data

7.53 s $\pm$ 0.06 s
7.38 s $\pm$ 0.06 s
7.47 s $\pm$ 0.06 s
7.43 s $\pm$ 0.06 s

Numerically:

$\bar{t}_{5,est} = 7.45250$ s

$\sigma_{est} = 0.0634429$ s

$\sigma_{est} = 0.06$ s

A Digital Instrument

Propagation of Errors

$z = x + y$
$z = x - y$ $\quad \Delta z = \sqrt{\Delta x^2 + \Delta y^2}$

$z = x * y$
$z = x / y$ $\quad \frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$

$z = A\,x \quad \Delta z = A\,\Delta x$

$z = x^n \quad \Delta z = |\,n\,x^{n-1}\,\Delta x\,|$

Repeated Measurements

- Repeated **n** times
- Each individual measurement has an error of precision $\Delta \mathbf{x}$

$$\Delta \bar{x}_{est} = \frac{\Delta x}{\sqrt{n}}$$

Significant Figures

- Discussed in Section 1.9 of Knight Ch.1
- Rules for significant figures follow from error propagation
  - Assume error in a quoted value is half the value of the last digit.
  - Errors should be quoted to 1 or 2 significant figures
  - Error should be in final displayed digit in number.
- Example: If a calculated result is: (7.056 ± 0.705) m, it is better to report: (7.1 ± 0.7) m.