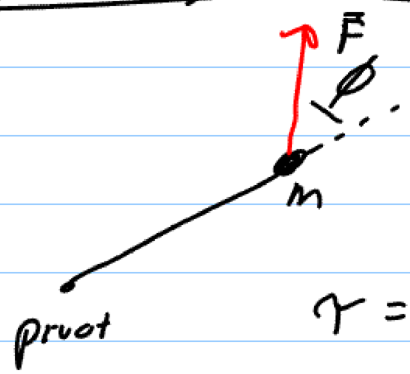
PHY138 Mechanics - Class 13 - Oct 25/06

§13.4 Dynamics



$F = ma$

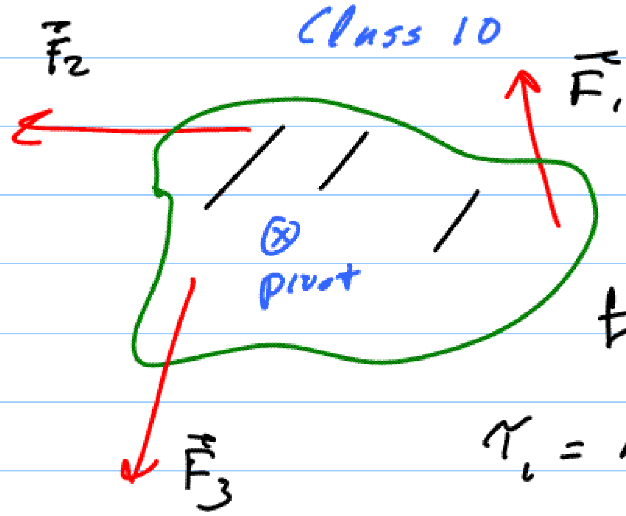
$F_t = m a_t$

$\tau = r F_t = m r a_t$

$a_t = \alpha r$

$\tau = mr^2 \alpha \leftarrow$

Class 10



RIGID BODY

for each mass m_i

$\tau_i = m_i r_i^2 \alpha$

$\tau_{tot} = \sum \tau_i = \left(\sum m_i r_i^2\right) \alpha$

3rd Law: 2 masses j & k

$$\tau_{j \text{ on } k} = -\tau_{k \text{ on } j}$$

internal torques cancel

$$\tau_{net} = \underbrace{\tau_1 + \tau_2 + \tau_3}_{\text{external torques}} = \left(\sum_i m_i r_i^2\right)\alpha$$

$$\tau_{net} = \left(\sum m_i r_i^2\right)\alpha$$

"Moment of Inertia" I

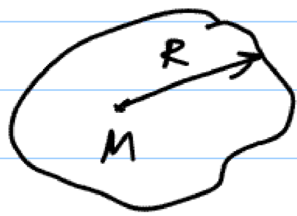
$$\tau_{net} = I\alpha$$

Linear: $a = \frac{F}{m}$

Rotational: $\alpha = \frac{\tau}{I}$

$$I = \sum_i m_i r_i^2 = \int r^2 \, dm$$

continuous body



$$I = c\, MR^2$$

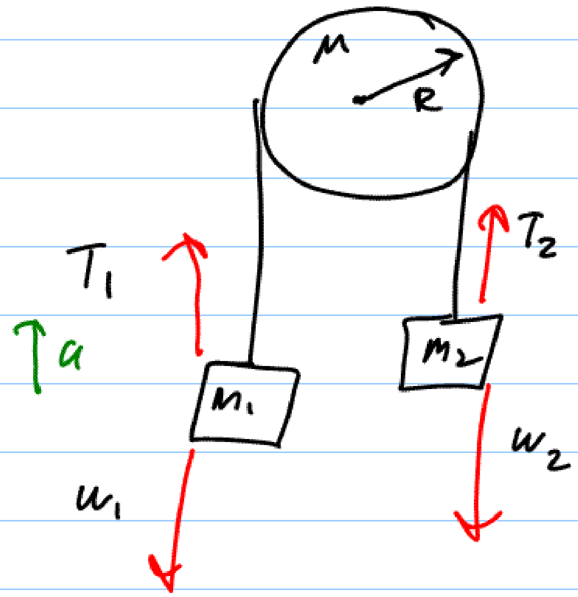
$c = 0$: point particle

$c = 1$: all mass is R away from axis of rotation

$0 < c < 1$: rigid body

particle model not appropriate

§ 13.5 - Rotation About an Axis



string massless
axle of pulley frictionless

$m_2 > m_1$

$\underline{M = 0}$

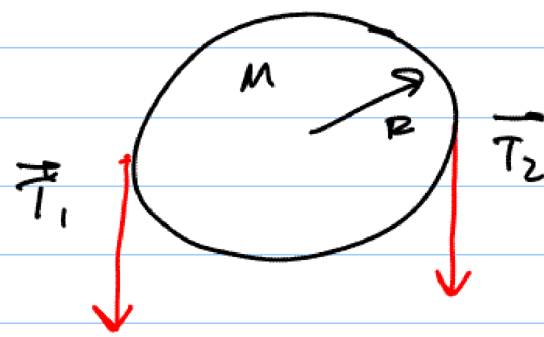
$a \downarrow$

$$\boxed{\begin{cases} m_2 g - T_2 = m_2 a \\ T_1 - m_1 g = m_1 a \end{cases}}$$

$$T_1 = T_2$$

$$a = \frac{(m_2 - m_1) g}{(m_1 + m_2)}$$

$\underline{M \neq 0}$



$$\tau_{net} = I \alpha$$

$$r T_2 - r T_1 = I \alpha$$

$$\boxed{r(T_2 - T_1) = c M R^2 \alpha}$$

$$\boxed{\alpha = a/R}$$

$$\boxed{a = \frac{(m_2 - m_1) g}{(m_1 + m_2 + cM)}, \text{ down}}$$

§ 13.6 - Equilibrium

$$\left.\begin{array}{ll} & \vec{F}_{net} = 0 \\ \text{add} & \tau_{net} = 0 \end{array}\right\}$$

At equilibrium! evaluate τ about any point

Rotations. must evaluate τ about the axis of rotation

Forces on LEG (notes)

$$\left.\begin{aligned} F_x &= 0 \\ F_y &= 0 \\ \tau &= 0 \end{aligned}\right\}$$

approximately true for walking

$$\left\{\begin{aligned} F &= 1.6\,W \\ R &= 2.4\,W \\ \phi &\sim 13^\circ \end{aligned}\right.$$

<u>Cane</u>: force $\frac{W}{6}$ $\boxed{\text{same side}}$!

Same as walking except

$$N = \frac{5}{6}W$$

$$F = 1.3\,W$$

$$R = 2.0\,W$$

$\boxed{\text{opposite side}}$!

$$F = 0.6\,W$$

$$R = 1.3\,W$$

Rot Energy

$K_{trans} = \frac{1}{2} m v^2$ ✓

$K_{rot} = \frac{1}{2} I \omega^2$ ✓

$U_g = M g y_{cm}$

$\vec{\omega}$ vector !