Class 15 - November 2/05
Laboratory Topic: Error Analysis

## 2 Kinds of Statements

(1) Exact! $2+3=5$

math

$\vec{p} = m\vec{v}$

defn

(2) Approximate!

$F_{spring} = -k\Delta s$

physical law

$g = 9.80\, m/s^2$

any quantity about real world

Example: measure period $T$
of pendulum

method: measure time
for 5 oscillations
$t_5$

$$T = \frac{t_5}{5}$$

$t =$ ~~5.97 s~~ ? whoops! should start counting at zero!

$t = 7.53\ s$ ✓

$= 7.38\ s$

$$d = 15.2\ cm = \frac{1}{2} g t^2$$

$$\boxed{t \sim 0.2\ s}$$

- Carl Friedrich Gauss (~1800)

→ Astronomer known for his math.

- discovered that repeated measurements of the same thing produce a distribution of similar results.
- distribution shape is a fit to a histogram called a "Gaussian":

an exponential →

$$N(x) = A\, e^{\left[\frac{-(x-\bar{x})^2}{2\sigma^2}\right]}$$

$A$ = some constant
$e$ = 2.7182...

$\left.\begin{array}{l}\bar{x} = \text{mean or average } x \\ \text{"sigma"} \rightarrow \sigma = \text{standard deviation}\end{array}\right\}$ these numbers can be estimated from the data

idea: 68% of the times you do a measurement you will find $x$ between: $\bar{x}-\sigma$ and $\bar{x}+\sigma$.

- Many distributions in life have Gaussian shapes.
  ie) · repeated measurements
  · People's heights
  · Random walk

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Repeat msrmnt $n$ times

If $n < \infty$, can only estimate $\bar{X}$, $\sigma$

For reasonable values of $n$
know $\sigma$ to 1 or maybe 2
significant figures

$\boxed{\text{Error in each individual msrmnt is } \sigma}$

Reading Error

① Digital Instrument
$\pm \frac{1}{2}$ of last digits

$\Delta t_5 \sim 0.005s$

② Analog Instrument
Your guess

2 ERRORS: $\sigma$ ; Read Err.

Choose Largest of the Two as "THE ERROR" in each individual msrmnt

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Propagation of errors.

Let's find the mean of several measurements, all with known error. $x_1 \pm \Delta x$ , $x_2 \pm \Delta x$

Rules of propagating errors: sum rule, product rule, multiplying by exact constant, exponent rule.

Mean: $$\bar{x} = \frac{(x_1 \pm \Delta x) + (x_2 \pm \Delta x) + (x_3 \pm \Delta x) + \cdots \underset{\text{values}}{n}}{n}$$

$$\bar{x} = \frac{x_{sum}}{n}$$

error in sum: $$\Delta x_{sum} = \sqrt{\Delta x^2 + \Delta x^2 + \Delta x^2 + \cdots_{n \text{ values}}}$$

$$= \sqrt{n \Delta x^2}$$

$$\Delta x_{sum} = \sqrt{n}\, \Delta x$$

error in mean: $$\Delta \bar{x} = \frac{1}{n}\left(\sqrt{n}\, \Delta x\right) = \frac{\sqrt{n}}{n} \Delta x$$

constant

$$\boxed{\Delta\bar{x} = \frac{1}{\sqrt{n}}\,\Delta x}$$

## Significant figures.

- If we calculate a number using other approximate numbers the calculated value may have many digits.
- The error in calculated value determines number of these digits that matter, or signify most likely answer.

ie) drop an object from rest: $d = \frac{1}{2} g t^2$

$g = 9.80\ m/s^2$

$t = 1.2\ s$ ← $\underbrace{\text{2 sig figs.}}$

this rule comes from error analysis.

calculated. ⟶ $d = 7.056\ m$

$\boxed{d = 7.1\ m}$