PHY138 Mechanics Test

Problem Answers and Marking Scheme

General Principles

Right answers always get full marks.
There are other correct ways to solve almost all of the questions. Marks for method(s) other than shown here are apportioned consistent with the way the marks are awarded for the methods shown below.
Any wrong answer that is used correctly in the following parts incurs no further penalty.
Missing or wrong units in a final answer: subtract 0.5 marks for each occurence. Note that treating a *radian* as a true unit is not treated as an error.
No deductions for incorrect significant figures in a final answer.

Part A (10 marks total)

■ Number 1 (3 marks)

■ Method 1: Using conservation of energy

$$W = \Delta K$$

$$(m_2 - m_1)\, g\, d = \frac{1}{2}(m_1 + m_2)\, v^2 \qquad \text{(1 mark)}$$

$$v = \sqrt{\frac{2\,(m_2 - m_1)\, g\, d}{(m_1 + m_2)}} \qquad \text{(1 mark)}$$

$$v = 10.5\,\mathrm{m/s} \qquad \text{(1 mark)}$$

■ Method 2: Using Newton's Laws directly

$$m_2\, g - T_2 = m_2\, a$$
$$T_1 - m_1\, g = m_1\, a$$
$$T_2 = T_1$$

Therefore, adding the first two equations above eliminates the tensions and:

$$(m_2 - m_1)\,g = (m_2 + m_1)\,a \qquad \text{(1 mark)}$$

$$a = \frac{(m_2 - m_1)\,g}{(m_2 + m_1)}$$

Sinca **a** is constant:

$$v^2 = 2\,a\,d \qquad \text{(1 mark)}$$

$$v = \sqrt{\frac{2\,(m_2 - m_1)\,g\,d}{(m_1 + m_2)}}$$

$$v = 10.5\,m/s \qquad \text{(1 mark)}$$

▪ Number 2 (3 marks)

$$d = \frac{1}{2}\,a\,t^2 \qquad \text{(1 mark)}$$

The next equation may have been solved when the student did **Number 1** above. Then award 1 mark again for:

$$a = \frac{(m_2 - m_1)\,g}{(m_2 + m_1)} \qquad \text{(1 mark)}$$

$$t = \sqrt{\frac{2\,d}{a}} = \sqrt{\frac{2\,d\,(m_2 + m_1)}{(m_2 - m_1)\,g}}$$

$$t = 3.23\,s \qquad \text{(1 mark)}$$

▪ Number 3 (2 marks)

$$T_1 - m_1\,g = m_1\,a$$

or:

$$m_2\,g - T_2 = m_2\,a \qquad \text{(1 mark)}$$

where:

$$T_1 = T_2 \equiv T = \text{the answer}$$

and **a** may have been found in either **Number 1** or **Number 2** above.

$$T = 65.3\,N \qquad \text{(1 mark)}$$

▪ Number 4 (2 marks)

$$\omega = \frac{v}{r} \qquad \text{(1 mark)}$$

where **v** is the answer to **Part 1.**

$$\omega = 3\ \mathrm{sec}^{-1}$$

or:

$$\omega = 3\ \mathrm{radians/sec} \qquad \text{(1 mark)}$$

Part B (8 marks total)

■ Number 1 (3 marks)

$$F_{\mathrm{on}\,1} = T_1 - m_1\, g = \frac{dp_1}{dt} = m_1\, a + v\, \frac{dm_1}{dt} \quad \text{(1 mark)}$$

$$m_2\, g - T_2 = m_2\, a$$
$$T_1 = T_2 \equiv T$$

$$(m_2 - m_1)\, g = (m_1 + m_2)\, a + v\, \frac{dm_1}{dt}$$

$$a = \frac{(m_2 - m_1)\, g - v\, \frac{dm_1}{dt}}{(m_1 + m_2)} \qquad \text{(1 mark)}$$

Note that the rate of change of m_1 is negative:

$$\frac{dm_1}{dt} = -1.5\ \mathrm{kg/sec}$$

$$a = 4.32\ \mathrm{m/s}^2 \qquad \text{(1 mark)}$$

■ The "other" Number 1 (3 marks)

$$m_2\, g - T = m_2\, a$$

or:

$$T - m_1\, g = m_1\, a + v\, \frac{dm_1}{dt} \qquad \text{(2 marks)}$$

where **a** is the answer from to the previous question.

$$T = m_2\, g\ m_2\, a$$

or

$$T = m_1\, a + v\, \frac{dm_1}{dt} + m_1\, g$$

Either of the above equations yields:

$$T = 54.8\ N \qquad \text{(1 mark)}$$

■ Number 2 (2 marks)

$$\alpha = \frac{a}{r} \qquad \text{(1 mark)}$$

where **a** is the answer to the first Number 1 above.

$$\alpha = 1.23\ sec^{-2}$$

or:

$$\alpha = 1.23\ radians / s^2 \qquad \text{(1 mark)}$$

Part C (10 marks total)

■ Number 1 (3 marks)

$$W = \Delta K$$

$$(m_2 - m_1)\ g\ d = \frac{1}{2}\ (m_1 + m_2)\ v^2 + \frac{1}{2}\ I\omega^2 \qquad \text{(1 mark)}$$

$$\omega = \frac{v}{r} \qquad \text{(1 mark)}$$

$$(m_2 - m_1)\ g\ d = \frac{1}{2}\ (m_1 + m_2)\ v^2 + \frac{1}{2}\ I\left(\frac{v}{r}\right)^2$$

Note: some students assumed that the pulley had uniform density, in which case I = 36.75 kg m^2, instead of the given 63 kg m^2. The problem wording made no such statement. Nonetheless do not deduct marks for this incorrect assumption. Below answers using this value of I are in blue and surrounded by brackets [xxx].

$$v = \sqrt{\frac{2\ (m_2 - m_1)\ g\ d}{m_1 + m_2 + I / r^2}}$$

$$v = 9.09\ m / s$$

$$[v = 9.62\ m / s] \qquad \text{(1 mark)}$$

■ Number 2 (3 marks)

Just as for **Number 1** above since **a** is constant:

$$v^2 = 2\ a\ d \qquad \text{(1 mark)}$$

$$a = \frac{v^2}{2d}$$ (1 mark)

$a = 2.43\ m/s^2$

$[a = 2.72\ m/s^2]$ (1 mark)

■ Number 3 (2 marks)

$\tau = I\alpha$ (1 mark)

$$\alpha = \frac{a}{r}$$

$$\tau = \frac{I\,a}{r}$$

$\tau = 43\ kg\,m^2/s^2$

$[\tau = 28\ kg\,m^2/s^2]$ (1 mark)

■ Number 4 (2 marks)

$L = m_1 v r + m_2 v r + I\omega$ (1 mark)

In the above, note that **r** is the radius of the pulley, and **v** is the answer to **Number 1**.

$$\omega = \frac{v}{r}$$

$$L = m_1 v r + m_2 v r + I\frac{v}{r}$$

$$L = (m_1 r + m_2 r + I/r)\,v$$

$L = 641\ kg\,m^2/s$

$[L = 606\ kg\,m^2/s]$ (1 mark)