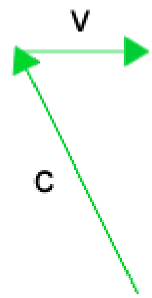
Problem Solutions

Problem 1

Part A

For the light traveling to Reflector R_1, the light that reaches the reflector must be going "upwind" relative to the interferometer, as shown. Therefore, its speed relative to the interferometer is



$$\sqrt{c^2 - v^2}$$

The light travels a total distance 2*L*, so the time is

$$t_1 = \frac{2L}{\sqrt{c^2 - v^2}} \tag{1.1}$$

For the light traveling to Reflector R_2, when it is traveling towards the reflector its speed is $c + v$ relative to the interferometer¸ and when it is returning to the beam splitter its speed is $c - v$. Thus the total time is:

$$t_2 = \frac{L}{c+v} + \frac{L}{c-v} = \frac{2Lc}{c^2 - v^2} \tag{1.2}$$

From Equations 1.1 and 1.2 it is fairly simple to show the Equation A.1 is true.

Part B

The difference in times is

$$t_2 - t_1 = \frac{2Lc}{c^2 - v^2} - \frac{2L}{\sqrt{c^2 - v^2}} = 2L\left[\frac{c}{c^2 - v^2} - \frac{1}{\sqrt{c^2 - v^2}}\right] \tag{1.3}$$

Evaluating this can be a bit of a challenge for cheap calculators because the two terms in the square brackets have almost the same value. Here is a form which may work better for your calculator

$$t_2 - t_1 = \frac{2L}{\sqrt{c^2 - v^2}}\left[\frac{c}{\sqrt{c^2 - v^2}} - 1\right] = 3.33\times10^{-16}\ \text{s} \tag{1.4}$$

Alternatively we can use the fact that $(1-x)^n \approx 1-nx$ for small x to show that the time difference is approximately

$$t_2 - t_1 \approx \frac{Lv^2}{c^3} \approx 3.33\times 10^{-16}\,\text{s}$$

In terms of the phase difference, this is

$$\Delta\phi = 2\pi \frac{t_2 - t_1}{T} = 2\pi\, f\,(t_2 - t_1) = 2\pi \times 0.200\,\text{rads} \qquad (1.5)$$

Part C

This is just all of the above except that the labels *1* and *2* are interchanged. Thus we can immediately write the answer:

$$\Delta\phi = -2\pi \times 0.200\,\text{rads}$$

Part D

Recall that when the phase difference is π we have complete destructive interference. Here we have gone from a phase difference of 0.400 π through 0 (constructive interference) to -0.400 π radians. This should be observable.

Problem 2

If the period of the source is T relative to Pablo, then the wavelength of the wave relative to him is

$$\lambda = (c+v)T \qquad (2.1)$$

But this period is the time dilated value. If the period of the source in a frame stationary relative to the source is T_0, then

$$T = \gamma T_0 = \frac{1}{\sqrt{1 - v^2/c^2}}T_0 \qquad (2.2)$$

The wavelength of the wave in a frame stationary relative to the source, λ_0, is related to this period by

$$T_0 = \frac{\lambda_0}{c} \tag{2.3}$$

Thus Equation 2.1 becomes

$$\lambda = (c+v)\frac{1}{\sqrt{1-v^2/c^2}}\frac{\lambda_0}{c} = \sqrt{\frac{1+v/c}{1-v/c}}\lambda_0 \tag{2.4}$$

Problem 3

Let's start with the value of zero. This is just the value if a signal propagating at *c* travels from Event 1 to Event 2. Although this may be obvious, let's do the math

$$c^2\Delta t^2 - \Delta x^2 = 0 \tag{3.1}$$

Divide by Δt^2.

$$c^2 - \left(\frac{\Delta x}{\Delta t}\right)^2 = 0 \tag{3.2}$$

But $\frac{\Delta x}{\Delta t}$ is just the speed of a signal propagating from Event 1 to Event 2, *v*, so:

$$v = c \tag{3.3}$$

If the interval squared is greater than 0, Eqn 3.2 becomes:

$$c^2 - \left(\frac{\Delta x}{\Delta t}\right)^2 > 0 \tag{3.4}$$

So

$$v < c \tag{3.5}$$

This makes sense. If the two events occur at the same place at different times, the interval squared is positive.

If the interval squared is less than 0, we get

$$v > c \tag{3.6}$$

This also makes sense. If the two events occur simultaneously at different places, the interval squared is less than 0, and a signal that connects them would have to move at infinite speed.

Problem 4

Part 1

If $u_{\mathrm{Lou}} = v$, then f is 1. This is reasonable: the particle doesn't move relative to the train and the light meets it at the rear of the train.

Part 2

If $u_{\mathrm{Lou}} = c$, then f is 0. This too is reasonable: the "race" ends in a tie.

Part 3

For Sue, the speed of light is $(c - v)$ when it is traveling to the front of the car and $(c + v)$ after it is reflected. Thus the equivalent of Eqn C.4 is

$$u_{Sue}(t_{1,Sue} + t_{2,Sue}) = (c - v)t_{1,Sue} - (c + v)t_{2,Sue} \qquad (4.1)$$

Therefore

$$\frac{t_{2,Sue}}{t_{1,Sue}} = \frac{(c - v) - u_{Sue}}{(c + v) + u_{Sue}} \qquad (4.2)$$

For Sue, the equivalents of Eqns. C.5 and C.6 are

$$\begin{aligned} (c - v)t_{1,Sue} &= L_{Sue} \\ (c + v)t_{2,Sue} &= fL_{Sue} \end{aligned} \qquad (4.3)$$

Eliminate L_{Sue} , solve for f and use Eqn. 4.2 and we get

$$f = \frac{(c + v)}{(c - v)} \frac{(c - v - u_{Sue})}{(c + v + u_{Sue})} \qquad (4.4)$$

Equating this to Eqn. C.7 we end up with

$$u_{Sue} = u_{Lou} - v \qquad (4.5)$$

This is the "common sense" result gotten by Galileo.

Problem 5

Energy is the time component of the vector 4-momentum. This answer is just from looking at the structure of the equation.