PHY132S – Relativity
Summary – 6 Classes on 3 Pages

The *Special* Theory of Relativity considers observers in uniform relative motion. The *General* Theory considers observers in any state of relative motion. Einstein's Equivalence Principle of 1908 means that the theory is also a theory of gravitation.

Both theories require a careful examination of how we determine where and when an event occurs relative to some inertial reference frame. We imagine a lattice of meter sticks and synchronized clocks. The meter sticks determine where an event occurs. We measure the time as read by a clock right beside where the event occurs: later we used the word *local* for this idea of restricting ourselves to time measured by a clock right beside the event.

Special Relativity

We discussed two threads in the development of the Special Theory:

1. Einstein: the laws of Physics, including Electricity & Magnetism, must be true in all inertial reference frames
2. Michelson-Morley: the speed of Earth relative to the ether is experimentally found to be zero.

These lead to the conclusion: **the speed of light has the same value for all observers.**

In some sense, this one statement is the entire Special Theory. Everything else is a consequence of this statement.

One immediate consequence is that the notions of Absolute Space and Absolute Time, or equivalently the idea of a medium for light, the ether, are superfluous.

From the constancy of the speed of light we derived that clocks that are moving relative to us run slowly.

The derivation involved *light clocks*, but we then realized that we were actually making a statement about time.

If $\Delta\tau$ is the time between two events measured by a clock where the events occurred at the same place, then a clock moving at speed v relative to this clock will measure a time interval given by:

$$\Delta t = \frac{1}{\sqrt{1-v^2/c^2}}\Delta\tau = \gamma\,\Delta\tau$$

In applying this equation, figuring out whether to multiply or divide by γ you need to remember only two things:

1. The cosmic ray muons live long enough to reach the Earth: theirs clocks are running slowly relative to an Earth clock
2. In the Twin "Paradox", the astronaut twin ends up younger.

From time dilation we showed that objects that are moving relative to us have their lengths along their direction of motion contracted by a factor $\sqrt{1-v^2/c^2}$.

The text shows that the constancy of the speed of light leads to problems with the idea that two events that are simultaneous for one observer will be simultaneous for all observers. In class we showed that the relativity of simultaneity follows from the relativistic length contraction.

Although the time and distance between two events is different for different observers the quantity $s^2 = (c\Delta t)^2 - (\Delta x)^2$ has the same value. This led some conclusions:

Time is the 4th dimension.
We should measure time in the same units as the three spatial dimensions.
The speed of light c is only a conversion factor for units.

We introduced spacetime diagrams as a visualisation technique for relativity.

We derived the relativistic addition of velocities directly from the constancy of the speed of light.

We considered inelastic collisions of objects moving close to c and realized there was a problem with Newtonian ideas about momentum and energy. We ended up realising that for an object with mass m moving at velocity $\vec{u}$ relative to us:

$$\vec{p} = \frac{1}{\sqrt{1-u^2/c^2}} m\vec{u}$$

Similarly, we were led to the statement:

$$E = \frac{1}{\sqrt{1-u^2/c^2}} mc^2$$

The energy has two components:

$$E = E_0 + K$$

where K is the kinetic energy.

We “read” the equation $E_0 = mc^2$ to realise that energy and mass are just different names for the same thing. An example of energy converting to mass and vice versa is pair production and annihilation.

General Relativity

“Spacetime tells mass how to move;
mass tells spacetime how to curve.”
-- Taylor and Wheeler

The theory is built from three pieces:

1. Geometry is Physics (Riemann, late 19^{th} century)
2. Inertia here is due to gravitational mass there (Mach, late 19^{th} century)
3. Locally acceleration is equivalent to gravitation (Einstein, 1908)

The theory predicts:

- Clocks in gravitational fields run slowly. We did a qualitative derivation using only the Equivalence Principle. This prediction is experimentally confirmed.
- Stellar aberration. This prediction is experimentally confirmed.
- The advance of the perihelion of Mercury. This prediction is experimentally confirmed although there are large errors in the data.
- Gravity waves. Experiments are currently under way to try to “see” gravity waves.
- Black holes. There are many excellent candidates for black holes.
- Expanding universe. We spent some time on Hubble’s 1926 data confirming this prediction.

We then discussed some cosmology, “open” and “closed” universes, and the big bang.

Up to about 15 years ago this was then end of the story. Physicists thought that we had a good understanding of cosmology. There were some small problems, including the flatness problem, the homogeneity problem, and the apparent existence of dark matter.

Then, in 1998 Saul Perlmutter and his colleagues did a Hubble-type experiment with supernovae that shows that the rate of expansion of the universe is increasing. The experiment has since been replicated by NASA. This makes no sense. Perhaps we are seeing the collapse of the General Theory.