Investigating the Relation Between the Period and the Moment of Inertia by Determining the Torsion Constant of A Wire

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The moments of inertia I and the period for one oscillation T of four different objects were measured and calculated. Using the linear relationship between T^2 and I, a graph of T^2 versus I was plotted to determine the slope, which is proportional to the torsion constant κ of a given wire. The torsion constant κ of the given wire was determined to be $3.8 \times 10^{-2} \pm 2.7 \times 10^{-3}$ kg·m²/s².

Introduction

The purpose of the torsion pendulum experiment is to determine the torsion constant κ for a given wire. The relation between the period T and the moment of inertia I of the oscillation of an object hanging from the wire is given by:

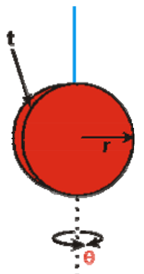
Equation 1: $$T = 2\pi\sqrt{\frac{I}{\kappa}}$$ which can be rewritten as Equation 2: $$T^2 = \left[\frac{4\pi^2}{\kappa}\right] I$$.

By measuring the period T for a number of objects with different moments of inertia I, a plot of T^2 versus I can be made. Since T^2 is proportional to I, the plot should be a straight line with a slope of $4\pi^2 / \kappa$. Thus, the torsion constant κ can be determined for the wire. The equations for moments of inertia for the objects used are given in Figure 1.

Observations

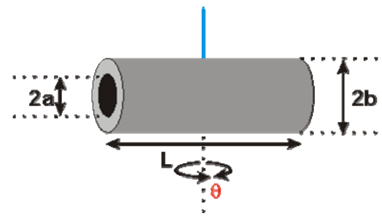
The objects whose moments of inertia were calculated include a solid brass disc, a solid metal cylinder and a hollow metal cylinder. The mass and dimensions of each object were measured using a balance and a ruler respectively, noting the reading errors. The objects were hung on the wire mounted at the top of the chamber (kept at constant length) in four different conformations (as seen in Figure 1). The Plexiglas door of the chamber was kept closed during measurements to prevent air currents from disturbing the motion of the object. Figure 2 illustrates a diagram of the apparatus. The same gentle push was given to each object to generate a torque so that they exhibit simple harmonic motion (angles ≤ 10˚). A stopwatch was used to measure the time for twenty oscillations of each object over five trials, noting the reading errors.

Example 1: A disc of mass *M*, radius *r* and thickness *t* oscillating around the diameter that goes through the center of mass.



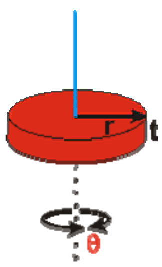
$$I = M\left(\frac{r^2}{4} + \frac{t^2}{12}\right)$$

Example 4: A hollow cylinder of mass *M*, length *L*, inner radius *a* and outer radius *b*.



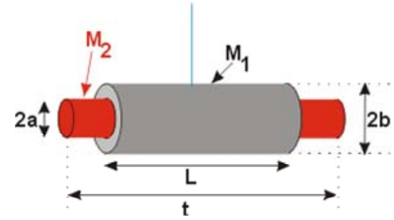
$$I = M\left[\frac{L^2}{12} + \frac{a^2+b^2}{4}\right]$$

Example 2: The same disc as before, but oscillating around the perpendicular to its face through the center.



$$I = \frac{Mr^2}{2}$$

Example 5: A hollow cylinder of mass *M1*, length *L*, inner radius *a* and outer radius *b* combined with a cylinder of mass *M2*, radius *a* and length ***t*.**



$$I = M_1\left[\frac{L^2}{12} + \frac{a^2+b^2}{4}\right] + M_2\left(\frac{a^2}{4} + \frac{t^2}{12}\right)$$

Figure 1. The equations of moments of inertia for the four objects used. [1]

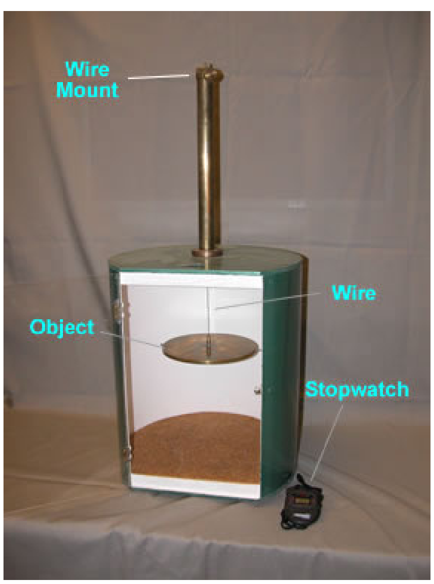


Figure 2. The apparatus used to measure the objects' period of oscillation. [1]

Analysis

Using the mass and dimensions of the objects, the moments of inertia (*I*) were calculated with the equations in Figure 1 for the four examples. The average time for twenty oscillations (20*T*) of each object over five trials was calculated by adding up the times and dividing the sum by five. The average period (T) for one oscillation was then calculated by dividing the average time for twenty oscillations of each object by twenty. Standard errors were calculated using known error formulae. Table 1 lists the final values used in plotting the graph T^2 versus I. Using the lab computer Faraday, a plot of T^2 versus I was created, where I was chosen as the independent variable and T^2 as the dependent variable, with errors dI and dT^2 respectively, as illustrated by Figure 3. Since T^2 is proportional to I, the plot should be a straight line. Using the least-squares fit of the data to a linear function, the slope of the line of best fit was determined to be $A_1 = 1039 \pm 74$. Since the slope of the line is equal to $4\pi^2 / \kappa$ as given by Equation 2, the value of the torsion constant for the wire κ was determined, noting the standard error.

Object	**Moment of Inertia, *I* (kg·m²)**	**Period For One Oscillation, *T* (s)**
Hollow + Solid Cylinder	$3.617 \times 10^{-3} \pm 3.5 \times 10^{-5}$	1.990 ± 0.0001
Hollow Cylinder	$1.659 \times 10^{-3} \pm 3.4 \times 10^{-5}$	1.339 ± 0.0001
Vertical Disc	$2.176 \times 10^{-3} \pm 3.3 \times 10^{-4}$	1.581 ± 0.0001
Horizontal Disc	$1.087 \times 10^{-3} \pm 1.3 \times 10^{-5}$	1.159 ± 0.0001

Table 1. Final values used to plot the graph T^2 versus I.

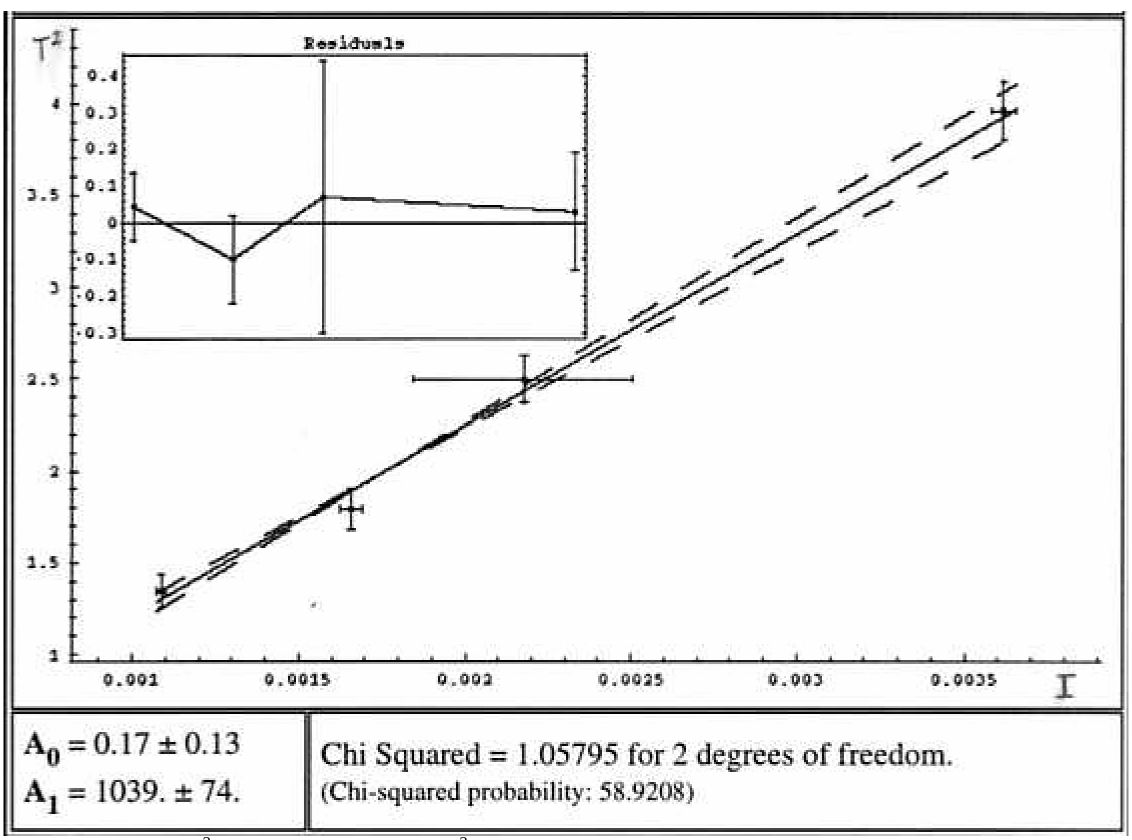


Figure 3. The plot of T^2 versus I with errors dT^2 and dI respectively, fitted with a least-squares fit. The slope of the line of best fit is determined to be $A_1 = 1039 \pm 74$.

Different objects have different moments of inertia since the mass is distributed differently about the axis of rotation. Consequently, the objects hanging from the wire would also have different periods of oscillation. However, different values of I and T share a common factor of κ if the objects are hung from the same wire at a constant length, as given by Equation 1 and 2. Using this relationship, the torsion constant κ of the given wire was determined to be $3.8 \times 10^{-2} \pm 2.7 \times 10^{-3}$ kg·m²/s², which is a reasonable value for a metal wire. The line of best fit, as illustrated by the plot on Figure 3, fit the data quite well within the standard error bars, confirming the linear relationship between T^2 and I. However, this experiment can be improved by measuring the period for twenty oscillations in more than five trials, to further illustrate the relationship between the period and the moment of inertia by minimizing error. More data points can be plotted to emphasize this relationship by calculating T and I for other objects on the same wire.

Conclusion

The torsion constant κ of the given wire was determined to be $3.8 \times 10^{-2} \pm 2.7 \times 10^{-3}$ kg·m²/s², which is a reasonable value for a metal wire. The relationship between the period and the moment of inertia as given by Equations 1 and 2 is confirmed by the linear relationship between T^2 and I, with a slope of $4\pi^2 / \kappa$. The main source of error in this experiment is reading error, especially in the measurement of the time for twenty oscillations.

References

1. Harrison, D. Torsion Pendulum Experiment. http://faraday.physics.utoronto.ca/IYearLab/Intros/TorsionPend/TorsionPend.html. Published October 2002. Accessed January 22, 2008.