

PHY110/138 Laboratory  
**Module C/D: Equilibrium and Oscillations**  
 November 6 – December 3, 2007

[Based in part on materials being planned for the *U of T Physics Practicals*, to be launched in Fall 2008]  
 Last revision: Nov. 2, 2007 by JJBH

## Purposes

1. To investigate the unstable equilibrium situation of a cart on a slope attached to a string under a given tension.
2. To investigate oscillations of a hanging mass about a stable equilibrium.

## Preparatory questions

Please answer the following two questions on a separate piece of paper. They may be done as homework and then the answers may be taped or stapled into your notebook as part of your write-up for Module C/D.

1. What is a simple pendulum?
2. Given a simple pendulum of length  $r$  and mass  $m$ , determine the oscillation period  $T$  if the perturbation angle  $\theta$  is very small (i.e.  $\theta < 10^\circ$ ). Show your reasoning.

## Equipment List

Item	Qty	Item	Qty
2.2 meter Track, with one bumper removed, Pasco ME-9453	1	Super-pulley with clamp, Pasco ME-9448A	1
Collision Cart, Pasco ME-9454	1	Metal Hoops, all of different diameters	4
White string	1 m	Stand with pivot for hanging hoops	1
PASCO Large Table Clamp, ME-9472	1	Tape measure	1
Metal rod, 60 cm long, 1/2" diameter	1	Digital stopwatch	1
Digital Angle Gauge, Matica B2646	1	Vernier caliper	1
		Digital scale	1/room

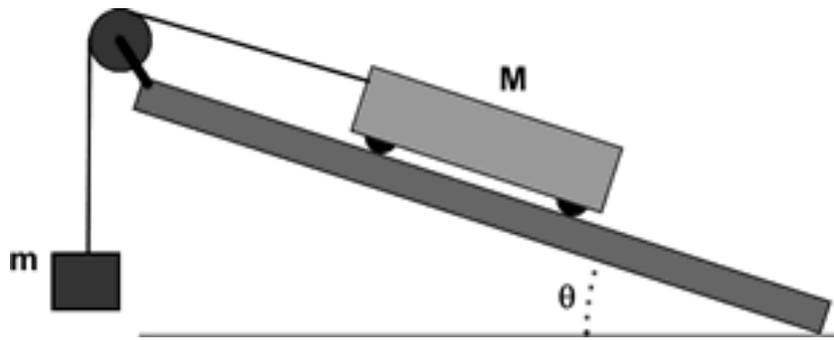
## Setup Notes

The track should be mounted with the ruler side toward where students sit. The bumper on the top that is closest to the end of the table should be removed and put into storage. The pivoting rod-clamp should go where the bumper normally goes. The rod clamp, large table clamp and aluminum rod can be used to secure one end of the track to the table. This allows that end to be raised. The super-pulley should be mounted on the end of the track in the centre so a string can be hung over it, attached to a dangling mass on one side, and a cart on the other. In fact, this can be done with 1 m of string.

## **Activity 1: Equilibrium of a cart being pulled by a constant force**

[based on Practicals 3.15 and 3.16]

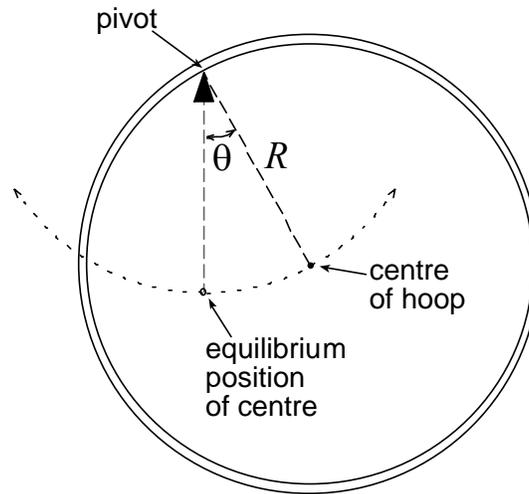
In the figure the Track is at an angle  $\theta$  with the horizontal. It is connected to a hanging mass  $m = 0.0500 \pm 0.0001$  kg by a massless string over a massless pulley.



- A. Use the balance to measure the mass  $M$  of the Cart.
- B. For some angle  $\theta$  the masses are in equilibrium, i.e. if they are at rest they remain at rest and if they are moving at some speed they continue moving at that speed. This is an unstable equilibrium, as there is no stable point that the cart will return to if perturbed. Predict the value of the angle  $\theta$  based on your best-known values for  $m$  and  $M$ . Express your result in degrees.
- C. The end of the Track that has the pulley mounted on it can be moved up and down using the attached clamp and the vertical rod mounted to the table. Verify your prediction of Part B. The digital angle gauge is a good way to measure the angle of the Track. How to use the digital angle gauge (Matica B2646):
  1. Turn on by pressing the ON/OFF button. A digital readout should appear. If this does not happen, consult your demonstrator or the Resource Centre.
  2. Find a surface and orientation of the gauge so that the bubble in the tube on top is centred. Press the ABS/ZERO button to zero the gauge.
  3. Place the gauge on the surface to be measured. Allow it to settle. The reading error is  $0.1^\circ$ .
- D. By how much can you change the angle  $\theta$  of the Track and not see any visible deviation from equilibrium?
- E. Express your results from Parts C and D by expressing the angle for equilibrium as  $\theta \pm \Delta\theta$ , with both values in radians.
- F. Imagine you are going to use this apparatus as a silly way of measuring the mass  $M$  of the Cart by measuring  $\theta$ . Recall that  $m = 50.0 \pm 0.1$  g. From Part E, what is the value and error of  $M$  determined this way? What is the dominant error in your measurements that has the greatest effect on your value of  $\Delta M$ ? [Note that if  $\theta < \sim 10^\circ$  you may use the small angle approximation  $\sin\theta \approx \tan\theta \approx \theta$ , valid when  $\theta$  is measured in radians.]

### **Activity 2: Oscillations of a hoop hanging from a pivot**

Consider a hoop with mass  $M$  and radius  $R$ . In the figure the hoop is supported by a pivot in a non-equilibrium situation. The centre of the hoop is not directly below the pivot, so gravity will produce a torque on the hoop which will rotate the hoop clockwise around the pivot, causing the centre to accelerate. The instantaneous position of the hoop is specified by the angle  $\theta$  that the centre makes with a vertical line dropped down from the pivot. Note that the angular acceleration  $\alpha$  is always opposite in direction to the displacement  $\theta$ . This is a stable equilibrium situation, as the system always tries to return to the equilibrium configuration (hoop hanging straight down) when perturbed.



- A. Measure the mass  $M$  and average radius  $R$  of the hoop. The best way to measure the average radius may be to find the diameter by measuring from the inside thickness of the hoop to the outside thickness at the other end. Divide the diameter by 2 to get the radius. Do this for several diameters just in case the hoop is not perfectly round. The error in  $R$  should include both your measurement uncertainty and the non-roundness of the hoop.
- B. When the maximum angle  $\theta_{\max}$  of oscillations is small ( $< \sim 10^\circ$ ), the hoop will exhibit Simple Harmonic Motion (S.H.M.). Make a prediction for the period for small oscillations  $T$ . Hints:
- The force of gravity produces a torque on the hoop which acts as if the application of the force is at the centre of mass of the hoop. For an angle  $\theta$  calculate the magnitude of the torque  $\tau$  on the hoop about the pivot point. Note that the magnitude of the torque is equal to the magnitude of the force times the lever arm, as given in Section 9.1 of Cutnell & Johnson (PHY110 text) or Section 13.3 of Knight (PHY138 text).
  - The moment of inertia of the hoop around the pivot is  $I = 2MR^2$ . The angular acceleration of the hoop is related to the torque by  $\alpha = \tau/I$ . Write down an equation for  $\alpha$  in terms of  $\theta$ .
  - The small angle approximation is  $\sin\theta \approx \tan\theta \approx \theta$ , where  $\theta$  is in radians. Use this to simplify your equation.
  - For S.H.M. to occur, the acceleration must be related to position by  $\alpha = -C_0\theta$  where  $C_0$  is some positive constant. The period of the motion is then given by  $T = \frac{2\pi}{\sqrt{C_0}}$ . Find an expression for  $T$  in terms of the physical properties of the hoop.
- C. Use a stopwatch to measure the oscillation period  $T$  of the hoop, and compare it with your prediction from Part B. One procedure would be to measure the time for 20 oscillations,  $t_{20}$ , and repeat the measurement 5 times. (This was suggested in part 10 of the error analysis assignment.)
- D. Repeat the measurements of  $R$  and  $T$  for three more hoops of various radii (4 in total). Each member of the team should perform a set of measurements of  $R$  and  $T$ . It may be useful to record the serial numbers of the hoops you use, in case you have to find the same hoops at a later time to repeat a measurement. Use the computer to fit a straight line to variables involving  $R$  and  $T$ . Extract a measurement for  $g$  from the slope and/or  $y$ -intercept of this fit. What is the dominant source of error in this determination of  $g$ ?