SIMILARITY

REFERENCES

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(2 weights)

INTRODUCTION: Laminar and turbulent motion, Reynolds number

There are many occasions in physics where it is not possible to perform experiments directly on the process under investigation. It then becomes necessary to perform model experiments. This is especially true in fluid dynamics where one may, for example, wish to determine the stresses on an airplane wing by performing model experiments in a wind tunnel.

In order to perform a successful model experiment one must have geometric similarity, similar boundary conditions and the ratios of the forces acting on the model must be the same as in the process modelled. The latter condition is taken care of by having the same similarity numbers in both cases.

For a spherical object, at low speed the flow is **laminar** : fluid displaced in the front of a moving sphere flows smoothly in regular patterns filling the vacuum left in the rear of the sphere. No eddies are formed. The resistance force exerted by a viscous fluid on a spherical object of radius r moving through it with constant velocity is given by Stokes' law

$$F_r = 6\pi\eta r v \qquad (1)$$

where F_r is the force that opposes the motion, η is the viscosity of the fluid and v is the constant velocity of the object. Stokes' law applies in a variety of situations like the movement of small particles suspended in a fluid (for example, the famous oil-drop experiment by R.A. Millikan). A sphere of radius r in free fall will reach its terminal velocity v_t when the force of the gravity F_g, reduced by Archimedean (buoyant) forces, balances the viscous force as derived by Stokes:
since

$$F_g = mg - \rho_f V g, \qquad (2)$$

equating F_r and F_g leads to

$$gV\Delta\rho = 6\pi\eta r v_t \qquad (3)$$

where g is the acceleration due to gravity, V is the volume of the sphere and $\Delta\rho$ is the difference between the densities of the object and fluid (ρ_o and ρ_f, respectively).

The flow is laminar only up to some critical flow speed relative to the solid boundary surface. Beyond this speed the flow becomes **turbulent**. The particles of the fluid begin to fluctuate in a random manner. Vortices are formed in the rear of the object moving through fluid, which result in a drag force, F_r, given by

$$F_r = 1/2 C_r \rho_f v^2 S \qquad (5)$$

(Historically, this is known as Newton's resistance law). S is the cross sectional area of the object exposed to the flow and $\boldsymbol{C_r}$ is **the resistance coefficient** (the other name is the drag coefficient).

The terminal velocity v_t of the free falling object can be obtained by equating F_g (2) and (3) and F_r (5):

$$gV\Delta\rho = 1/2 \rho_f C_r v_t^2 S \qquad (6)$$

The drag coefficient C_r is given by

$$C_r = \frac{2gV\Delta\rho}{v_t^2 S \rho_f} \qquad (7)$$

You should verify that C_r is dimensionless.

A flow may change from laminar to turbulent quite suddenly as conditions (velocity, boundary, etc.) are changed. In the transition region the flow is unstable and may fluctuate from one type to the other.

The nature of these conditions was studied by O.Reynolds in 1883. For an object moving steadily through a fluid, the Reynolds number, $\boldsymbol{R_e}$**, is the only dimensionless number needed to characterise the flow around the body**

$$R_e = \rho_f \frac{vd}{\eta} \qquad (8)$$

where v is the flow speed relative to the solid body exposed to the fluid, d is the model diameter (or other characteristic dimension), η is the viscosity of the fluid and ρ_f the density of the fluid. (In our experiment v is the terminal velocity v_t of an object - sphere or cylinder - falling in a liquid-filled tube). Physically the Reynolds number is the ratio of the inertial to the viscous forces acting on the body. Since the inertial force is dominant in turbulent flow, while the viscous force is dominant in laminar flow, large values of R_e are associated with turbulent flow and small values with laminar flow.

The Reynolds number of a system characterises the behaviour of a real systems; such systems can be studied using small scale models.
In our experiment, if $R_e < 80$ the fluid motion is substantially **laminar**. When $R_e > 2000$ the motion becomes **turbulent.**

The resistance coefficient C_r depends on a number of variables, but mainly on the Reynolds number. It is also a function of the objects shape. The C_r of a given model is typically constant in the turbulent regime, but becomes a function of the Reynolds number when R_e decreases. In the limit of the laminar regime C_r x R_e is a dimensionless constant which, from (3), (6) and (8), has a theoretical value of 24 in the case of a sphere.

Once the C_r versus R_e curve for a model has been determined it is possible to predict the terminal velocity of another object geometrically similar to the model in any other fluid medium. To do this we introduce one further similarity number

$$B_e = \left(\frac{2Vd^2 \Delta\rho}{S} \right) \left(\frac{g\rho_f}{\eta^2} \right) = C_r R_e^2 \qquad (9)$$

B_e, the best number, depends only on the model characteristics and fluid parameters**.**

EXPERIMENT

From your experiments, you will be able to plot C_r versus R_e and B_e versus R_e curves for each model (balls and cylinders).

The equipment consists of cylinders and tanks containing distilled water or a concentrated mixture of glycerin in water, precision hydrometers to obtain the densities, thermometers, various sizes of different teflon models (i.e. spheres and cylinders) and calipers; a stop watch, a polaroid camera and a photographic stroboscope for determining the fall velocities of the models.

Use a stop watch for slowly falling models. Use a polaroid camera and a photographic stroboscope for faster moving models (usually in pure water tanks). Adjust the frequency of strobe light before taking pictures (you should be able to obtain several images of the same falling object on one picture). Make sure that the ruler's scale is visible on your pictures.

1. Explore the region of small Reynolds numbers ($R_e < 10$) by using the smallest specimens of the models in the glycerin water mixture. First try to use the smaller teflon spheres in the concentrated mixture of glycerin and water. The spheres will be in the range of 1/16" to 1/2" or 3/4" in diameter. Note that although some six sizes are provided in this range, five or six different diameters will likely provide enough data for a meaningful interpretation of the Stoke's Law region.

2. You might try to determine several points in the turbulent region ($R_e > 2000$). The instability region $80 < R_e < 2000$ is not so meaningful since, as you will see, the results become less consistent due to random motion of the model. Obtain C_r vs R_e and B_e vs R_e curves for each model. Comment on the obtained $C_r(R_e)$ dependence. Then by calculating the B_e from (9) for any other medium (e.g., air) and any other geometrically similar object (e.g., a table tennis ball), you can obtain the R_e in that medium and thus its terminal velocity.

The Reynolds number between 10 and 100 will be difficult to explore due to container size limitations and/or fluid viscosities available. However for $R_e > 200$ the teflon spheres 1/16" to 2" diameter can be used in distilled water. Try 1/6", 1/8", 1/4", 1/2", 1 1/2", and 2". It will be necessary to use the smaller water tank for the smaller spheres to get a "light pipe" effect for the Xenon stroboscope and camera timing arrangement.

In order for the results of this experiment to be meaningful the measurements must be made precisely and with great care.

SUGGESTED GUIDELINES

There is a sufficient number of spheres and cylinders so that it is not necessary to recover them after a velocity determination is made. The technicians will remove the models from the tanks between laboratory periods.

In order to obtain the viscosity of the glycerine/water mixture you will measure the density of the mixture by using the hydrometers provided. Determine the viscosity using the chart of viscosity (measured in centipoises) versus density for glycerine/water mixture at various temperatures. Note that the SI units of viscosity are the $N\cdot s/m^2 = Pa\cdot s$. The equivalent cgs unit is the $dyne\cdot s/cm^2$, which is called poise (P). The following relation exists between the two units of viscosity:
1 poise (P) = 0.1 Pa·s. (Note that 1dyne=10^{-5}N).

The spheres manufactured in fractional inch sizes 1/16" through 3/4" are within ±0.002" diameter. For 1" and over the tolerance is ±0.005" for diameter. The density of the spheres should be determined (by weighing them and measuring their size).

The maximum diameter sphere that can be used in the glycerin/water mixture will be limited by wall effects and the finite length of the container. For the diameters of teflon cylinders use the same values as you used above for the spheres.
Read off the strobe frequency before each measurement.

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