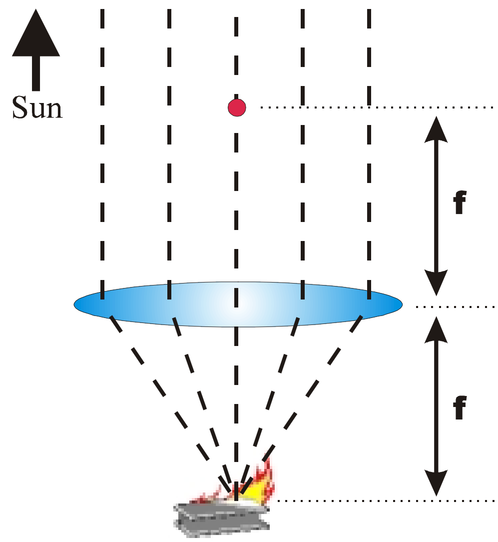
LENS OPTICS

INTRODUCTION

You may at some time have used a magnifying glass to set small pieces of paper or leaves on fire with sunlight. The sun is so far away that essentially all of its rays are parallel. The lens of the magnifying glass focuses all incoming light rays that are parallel with its axis at a single point, called the *focal point* of the lens. So what you were doing was concentrating all the sun's rays that entered the magnifying glass at one of the focal points of the lens. It turns out that lenses have two focal points, each a distance called the *focal length* **f** away from the centre of the lens.



The focal length of a lens depends on the radii of curvature of its surfaces and on the index of refraction of the material the lens is made from. This can be expressed, with some effort, by the *lens-maker's equation:*

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \ , \qquad (1)$$

where n is the index of refraction of the glass of the lens, and R_1 and R_2 are the radii of curvature of the front and back lens surfaces respectively.

Often instead of specifying the *focal length* of a lens one quotes the *power*, P=1/f, of the lens. *P* is measured in diopters. For example a lens of 2.5 diopters has a focal length of 0.4 m.

If all light rays parallel to the lens axis are directed to the focal point, the converse is also true: all rays from the focal point emerge from the lens parallel to the lens axis. If the lens is sufficiently **thin**, rays going right through the centre of the lens emerge unchanged. Three rays from an *object* are shown in the following figure: one leaves the object parallel to the lens axis, one goes through the centre of the lens, and the third goes through the focal point. All three rays then converge at the position labelled *real image*. In fact, it can be shown that *all* light rays leaving one position of the object and entering the lens converge at the corresponding position of the image. A small amount of geometry also shows that:

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f} \qquad (2)$$

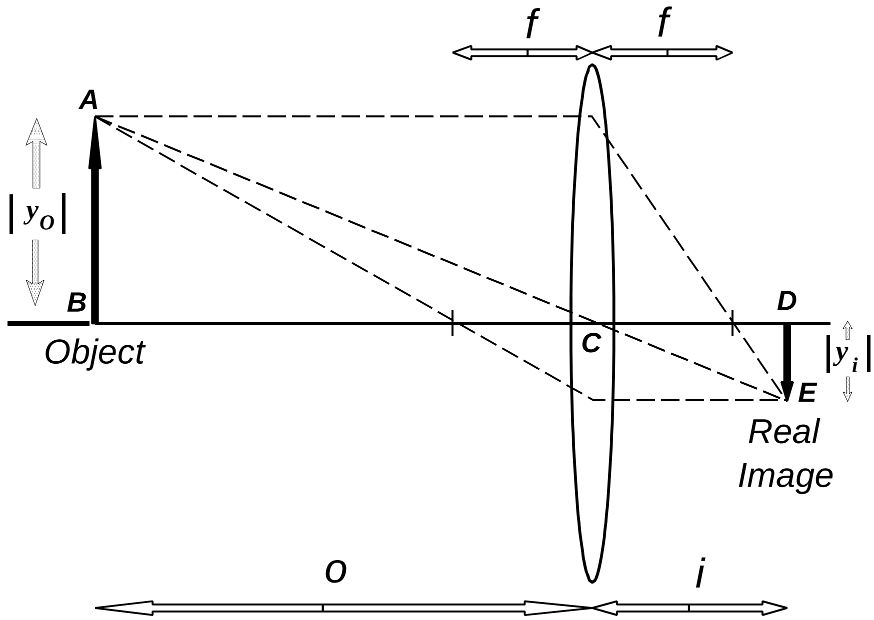


Figure 2. - Image formation with a thin lens.

Note that **o** and **i** are measured from the centre of the lens.

Simple geometry also shows that the magnification **M** of the lens is:

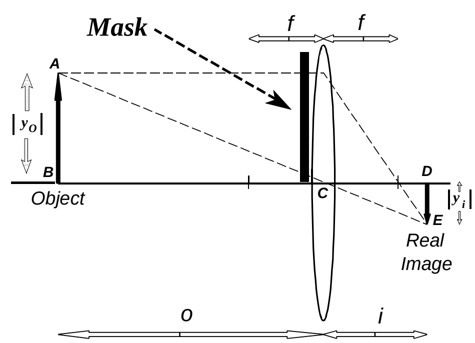
$$M = \frac{y_i}{y_o} = -\frac{i}{o} = \frac{1}{1 - o/f} \qquad (3)$$

where by convention **y** is positive if the arrow is upright, negative if upside down.

A real image cannot occur if the object's distance to the lens is less than the focal length (o < f). In such a case, one gets a *virtual* image, which can be seen with the naked eye but not displayed on a screen. This is how one typically uses a magnifying glass.

QUESTIONS

1. Suppose that the upper half of the lens was blocked out by a piece of paper as shown in the figure to the right. What would you predict would happen to the image? Think about your answer BEFORE you put the masking tape in place - is your prediction fulfilled? If not, can you explain why?



2. Keep the masking tape in place while you investigate the following phenomenon (that will make it easier on your eyes!) Set the equipment up so that you get a good sized image on the screen. Then remove the screen and try and locate the image in your eye; you will find that you have to move your head back and forward a bit to find a clear image. Is the position of the image on the screen at the same place as your eye when the image is focussed? Can you explain?

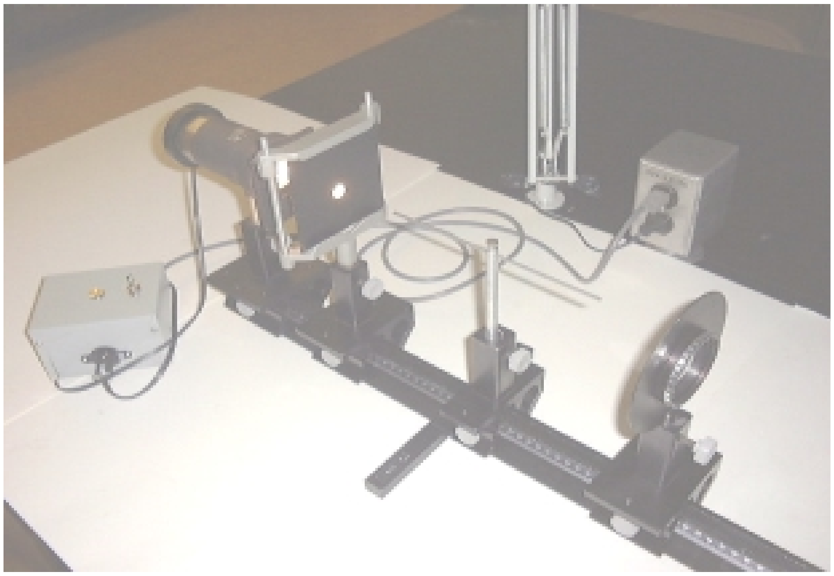
A SCENARIO

Many of the experiments in the I Year Physics Laboratory can have multiple *scenarios* to provide a motivation to the experiment. For example, you could imagine that the experiment is being done at some point in history before anything about the result was known. Alternatively, you might approach the experiment as an exercise to determine how well a particular apparatus can confirm a particular result. Often there are many different possible scenarios and we leave it to your imagination to supply the one that is most interesting to you.

We shall supply a scenario for the *Lens Optics* experiment. We imagine that you are working for the *XYZ Organisation*, and the apparatus that you are using will be used in a production environment of the organisation by a technician to make measurements of the focal length of a large number of lenses. The focal lengths will all have values fairly close to the one supplied with your apparatus. Your job is to design a procedure that provides the best measurement of the focal lengths of these lenses with the minimum effort by the technician.

EXPERIMENTAL PRELIMINARIES

Estimate the focal length of your lens.



You will see that the *optical bench* has a high quality metal tape rule affixed, which you will use to measure the positions of the object, lens, and image. The *holders* slide back and forth on the bench, and their position may be read by noting where one edge of the holder aligns with a reading on the metal rule; it is your choice as to whether you do your readings with the left or right side of the holder.

Of course, what you will really want to know is the distances between the object and the center of the lens, **o**, and the distance between the center of the lens and the image, **i**.

You will need to measure the thickness of the lens. **Caution**: try not to scratch the lens surface when you measure its thickness.

You are given an Aluminum rod whose length **L** is 20.00 ± 0.01 cm.

Adjust the position of the lens so that the left side of the rod is just touching the object and the right side is just touching the middle of the lens. You now know that the distance from the object to the center of the lens is 20.00 cm plus one half the thickness of the lens. **Caution:** don't break the paper of the object.

Read the positions of the object holder and the lens holder on the metal rule. If no correction factor were needed, the difference between these two numbers would be the already known distance from the object to the center of the lens. Calculate the correction factor for these measurements.

The image will be formed on the white screen held in place by its own holder. In order to know the distance from the center of the lens to the image you will need a second correction factor, which you should also determine.

THE FIRST PROJECT

As discussed above, the determination of the focal length **f** of the lens depends on the distance from the object to the center of the lens, **o**, and the distance from the center of the lens to the image, **i**, according to:

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

The determination of both **o** and **i** depend on:

- The readings on the scale of the optical bench of the position of the lens holder. The uncertainty in this number is probably just the reading error.

The value of **o** is the position of the lens holder minus the position of the object holder plus the correction factor. Thus the measurement of **o** also depends on:

- The readings on the scale of the optical bench of the position of the object holder. The uncertainty in the number is probably just the reading error.
- The correction factor that you determined with the 20.00 cm rod.

The value of **i** is the position of the image holder minus the position of the lens holder plus the other correction factor.

Thus the measurement of **i** also depends on:

- The readings on the scale of the optical bench of the position of the image screen holder. The uncertainty in the number is probably just the reading error.
- The second correction factor that you determined with the 20.00 cm rod.
- How much you can move the image screen holder back and forth on the optical bench without seeing any appreciable difference in the quality of the image.

The precision in your determination of **i** probably depends almost entirely on the last factor above. But, you can readily demonstrate that the amount of this "wiggle factor" depends on whether **i** is large or small. One may determine the focal length with: a large value of **o** and a small value of **i**; a small value of **o** and a large value of **i**; or medium values of both. Thus the precision in your determination of the focal length of the lens may depend on which of these three alternatives procedures is used.

☞ You are to determine which of the three possible procedures give the best value of the focal length of the lens.

In order to make the technician's measurement as simple and fast as possible, assume that the position of the object and lens holders are fixed. Thus the technician mounts the lens and moves the image screen to the position of the best focus and reads the position of the image screen holder.

☞ Assume that each lens is symmetric: its radii of curvature of the front and back lens surfaces, $\mathbf{R_1}$ and $\mathbf{R_2}$, are the same number. Does the value of the focal length depend on the thickness of the lens?

☞ Present the final formula, suitable for putting into a simple computer program, in which the inputs are the reading of the position of the image screen and perhaps the thickness of the lens; the output is the value of the focal length of the lens. Assume that the technician's vision is about as good as yours, so the uncertainty in **i** for the technician's measurements is about the same as yours. What is going to be the final uncertainty in the technician's measurement of the focal length?

DISPERSION

Often the index of refraction **n** depends on the wavelength λ of the light: this is called *dispersion*. From the lens-maker's equation (1), the focal length **f** of a lens made of dispersive glass will also depend on λ.

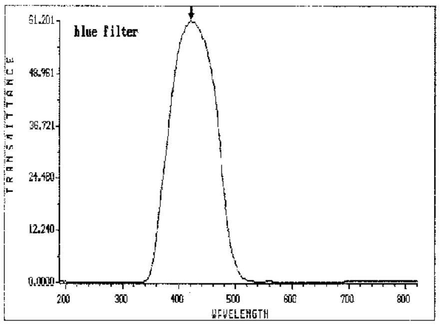
Thus, if white light is incident on the lens the different colours will be focused at different positions. This is called *chromatic aberration*.

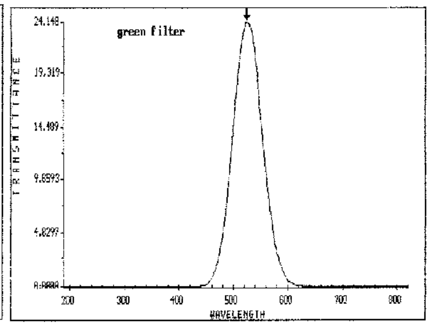
Now your supervisor at XYZ Organisation wants you to see whether this phenomenon is measurable with your apparatus and lens. You are supplied with color filters which mount on the light source behind the object. The maximum transmission of the filters are at wavelengths: red, 6900Å; green, 5300Å; blue, 4200Å

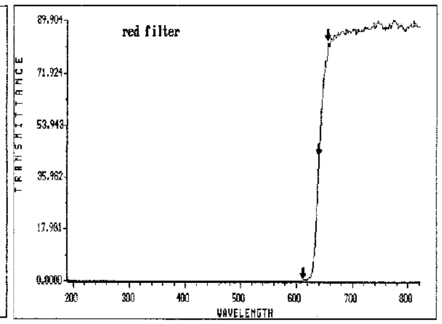
☞ What is your report to your supervisor?

Filter Transmission

Blue filter	Green filter	Red filter
arrow marker at 420 nm	arrow marker at 524 nm	arrow markers at 612, 640 and 656 nm







Major parts of this Guide Sheet were written by David Harrison, November 1999. Thanks to I Year Laboratory student Janet Null for the questions that prompted this Guide Sheet. Thanks to Doug Macintosh, Dept. of Chemistry, for measuring the transmission of the filters. Other parts of the history of this document are: (jv - 85,87, cp - 93, tk - 95, mf - 95, tk -98)