THE CAVENDISH EXPERIMENT

References

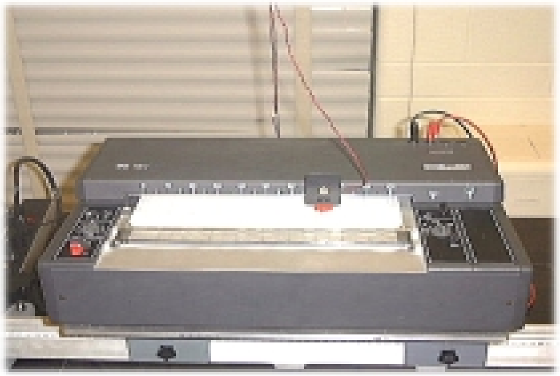
All references are available at the **R**esource **C**entre.

- J.W. Beams, "Finding a Better Value for G", Physics Today **24(5)**, 35 (May 1971).
- G.G. Luther and W.H. Towler, "Redetermination of the Newtonian Gravitational Constant G", Phys. Rev. Lett. **48**, 121 (1982).
- G.W. Fischer, J.L. Hunt and P. Sawatzky, "Automatic recording for the Cavendish balance", Amer. Jour. Phys. **55**, 855 (1987).
- Leybold, "Directions for use: Gravitation Torsion Balance".
- Extract from G.R. Noakes, *New Intermediate Physics*.

Apparatus notes

The apparatus consists of two main parts:

1. The Cavendish balance itself. The details of the balance may be found in the manufacturer's (Leybold) "Directions for use".



2. The strip chart recorder and associated electronics.

Details of the recorder and circuitry may be found in the paper by Fischer *et. al.* Briefly, a 2-element photodiode is mounted on top of the pen of the recorder. The output of the photodiodes are connected to the *Cavendish controller*, which outputs a voltage which is fed into the inputs of the recorder. Thus, the pen of the recorder will follow the laser beam reflected from the mirror of the Cavendish balance. The controller also contains a circuit which will cause the pen to seek for the beam.

Experimental notes

In this experiment, you will determine the equilibrium position of the reflected beam when:

1. the two large lead balls are not mounted on the frame of the Cavendish balance.
2. the two large lead balls are mounted and positioned fully clockwise on the balance.
3. the two large lead balls are mounted and positioned fully counter-clockwise on the balance.

You will determine these positions by analyzing the chart of the oscillations: it is not necessary to wait for the system to come to equilibrium. The chart will also allow you to determine the period of oscillation of the balance, and therefore the torsion constant of the wire support.

It will take about 45 minutes for you to take your data for each of these situations; Thus, you should *carefully* read this section and begin taking data as soon as possible.

Note: The apparatus may already be set up completely so you may start taking data immediately. Do not make any adjustments to the apparatus. If you have doubt, contact your demonstrator.

Then while the chart recorder is taking your first data you may read the following section on data analysis and look over the material in the references.

First insure that the balance is level, and that the inner frame holding the two small lead balls is swinging freely. The two knurled knobs under the balance support two pans which can be raised to arrest the motion of the balls: do not use these unless necessary.

The metal disc and thumb screw on the top of the balance adjusts the equilibrium position of wire supporting the inner frame. It should not need adjustment.

One difficulty of the experiment is aligning the laser beam. The incident and reflected beam **must** lie in a plane that is perpendicular to the axis of rotation of the balance. Further, the reflected spot must strike the photodiodes directly in the centre. One way to align the system is to leave the recorder off but with the pen down: then you may slide the recorder on the optical bench until the photodiodes are horizontally aligned with the beam. Now you may *gently* adjust the laser until the reflected beam strikes the photodiodes. In the course of making this adjustment, the balance will probably be disturbed and will start vibrating: wait a few minutes for it to settle down before proceeding. Check the alignment by turning on the recorder and the controller to see if the pen tracks the beam.

The control on the lower-right of the recorder labelled (roughly) <|**0**|> controls the zero-offset of the pen. Do not adjust once you start taking data. Similarly, do not adjust the laser between sets of data.

When the inner frame is oscillating back and forth you want to position the recorder so that the entire path of the oscillations is being tracked. A good chart speed when taking data is 1 cm/min. The recorder manufacturer claims a chart speed accuracy of ±70 ppm.

To get a chart off the recorder, do **not** try pulling the paper. Put the chart speed at its highest value and "fast forward" the paper. The serrated clear plastic blade on top of the recorder can then be used to tear off the paper.

Data Analysis

In Figure 1, the position labelled *E* is the equilibrium position of the balance with the heavy masses M not on their frame. When the large masses M are in position 1, the gravitational force F_g between *each* pair m and M is:

$$F_g = G \frac{Mm}{x_1^2} \quad [1]$$

where x_1 is the distance between the centres of m and M.

Thus, the *total* torque Γ_g on the inner frame is:

$$\Gamma_g = G \frac{Mm}{x_1^2} 2d \cos\alpha_1 \quad [2]$$

where 2d is the distance between the masses m. When the large masses M are in position 2, there is a similar equation with subscript 1 and replaced by 2.

At equilibrium, this torque is balanced by a torque from the wire supporting the inner frame. This torque is given by $k\theta_1$ (where k is the torsional constant and θ_1 is the angle of twist of the wire.), so:

$$G \frac{Mm}{x_1^2} 2d \cos\alpha_1 = k\theta_1 \quad [3]$$

The torsion constant can be determined by measuring the period T of oscillation as the frame approaches equilibrium.

$$T = 2\pi (I/k)^{1/2} \quad [4]$$

where I is the moment of inertia of the inner frame.

Assuming that the moment of inertia I of the inner frame is just $2md^2$, equations 3 and 4 can be combined to eliminate *m.*

If the equilibrium position E is symmetric, then $\theta_1 = \theta_2 = \alpha_1 = \alpha_2 \equiv \theta$. Also, the distance between the masses m and M ($x_1 = x_2 \equiv x$) can be shown by simple geometry to be:

$$x = R + w/2 - d\sin\theta \quad [5]$$

where w is the width of the case and d is the distance of m from the axis of rotation and equals 5.00cm.

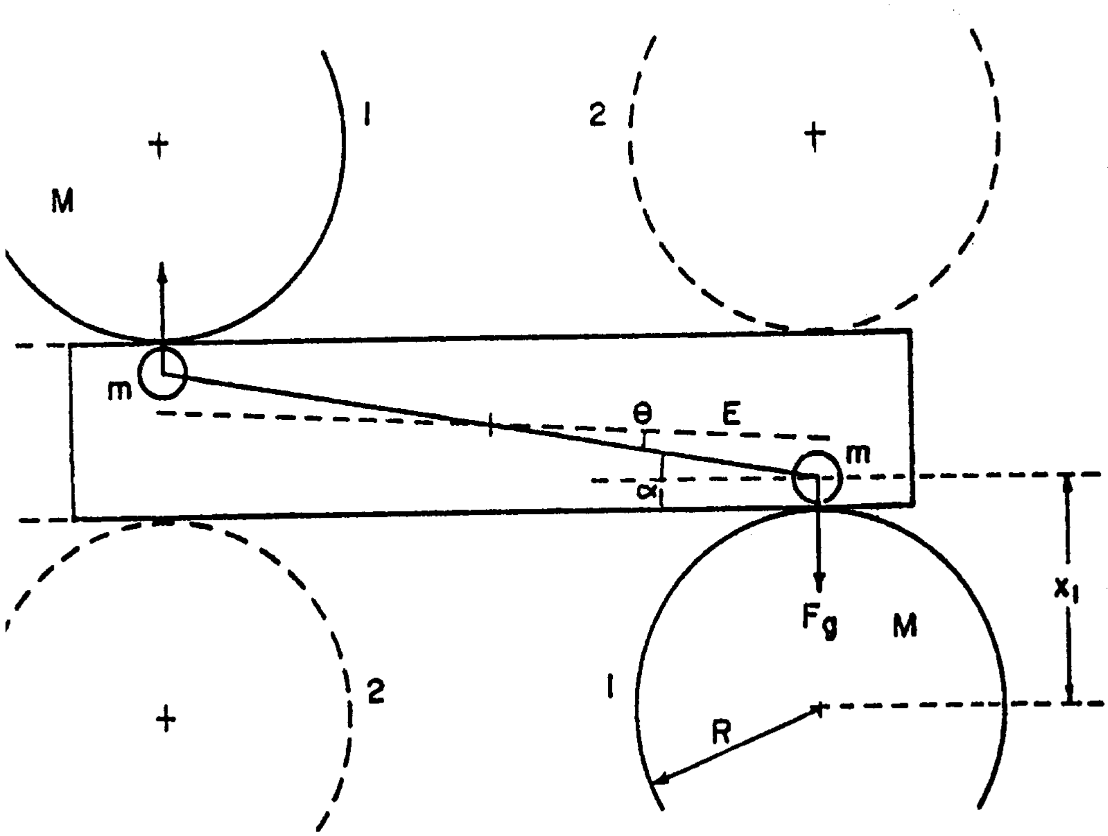


Figure 1. - Top View of Cavandish apparatus

A number of subtleties exist in the approximations made in the above analysis. An incomplete list includes:

- The correctness of the assumption that E is symmetric.
- The torsion pendulum is clearly damped harmonic motion. Is it *simple* harmonic motion? What effect does the damping have on the determination of the period?
- Each mass m will also be attracted by the remote second mass M. What effect does this have on your result?

In evaluating these effects, the crucial questions are:

> What is the dominant error in the determination of G? Does the approximation being made have an effect comparable to this dominant error? If yes, what can be done about it?

(D. Harrison, Sept 1987)