THE AIR TRACK (II)

References:
[1] The Air Track (I) - First Year Physics Laboratory Manual (PHY138Y and PHY110Y)
[2] Berkeley Physics Laboratory, 2nd edition, McGraw-Hill Book Company
[3] E. Hecht: Physics: Calculus, 2nd edition, Brooks/Cole 2000

Introduction

In Part 1 of this second series of experiments with the air track, you will investigate different types of collisions. The principles used to analyze the experimental data are: Newton's second and third laws which are used to develop the principle of conservation of momentum.

In Part 2 you will study a variety of dissipative (damping) forces. These forces include the various kinds of friction, magnetic damping, interaction forces in collisions which are not perfectly elastic, etc.

Part 3 is dedicated to periodic motion. Harmonic motion, damping and coupled oscillators are studied.

In order to get 2 weights, you have to do: Part 1 (all) and 2.1 (Viscous damping) from Part 2. To get 3 weights, you have to do Part 1 (all), Part 2 (all) and two experiments from Part 3.

1. Collisions

1.1 Theoretical background

We have two gliders (masses m_1 and m_2, velocities v_1 and v_2, respectively) moving on a horizontal track. We take v_1 to be positive if m_1 moves to the right and negative if to the left, and the same for v_2. Both v_1 and v_2 are functions of time.

When the gliders collide, they stay in contact for a very short time and they exert forces on each other. We assume that no other horizontal forces are exerted. Let the forces on m_1 and m_2 be F_1 and F_2, respectively, with the same sign conventions as for the velocities.

According to Newton's second law:

$$F_1 = m_1 \frac{dv_1}{dt} \qquad (1)$$

$$F_2 = m_2 \frac{dv_2}{dt} \qquad (2)$$

Newton's third law states that the two interaction forces have equal magnitude but opposite directions:

$$\vec{F}_1 = -\vec{F}_2 \qquad (3)$$

Combining (1), (2) and (3) we get:

$$m_1 \frac{dv_1}{dt} + m_2 \frac{dv_2}{dt} = 0$$

or:

$$\frac{d}{dt}\left(m_1 v_1 + m_2 v_2\right) = 0 \qquad (4)$$

The quantity $m_1 v_1$ is defined to be the momentum of the first glider, usually denoted by p_1, and similarly for the other glider.
Equation (4) states that the total momentum of the system p_1+p_2 does not change after the collision with respect to the initial value (its first derivative is zero). **In all cases where there are no external forces, the total momentum of the colliding objects is conserved.**
In an *inelastic collision* the final kinetic energy of the system is different from the initial one. All collisions between macroscopic objects are more or less inelastic. In *elastic collisions*, the kinetic energy of the system is conserved.
We shall take the simple case of glider m_1 (velocity v_0) colliding with glider m_2 at rest. After collision, velocities will be v_1 and v_2, respectively.
Let R be the ratio of final to initial kinetic energy of the system:

$$R = \frac{\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2}{\frac{1}{2}m_1 v_0^2} \qquad (5)$$

which becomes, for the particular case of equal masses:

$$R = \frac{v_1^2 + v_2^2}{v_0^2} \qquad (6)$$

Another useful quantity is the coefficient of restitution r:

$$r = \frac{v_2 - v_1}{v_0} \qquad (7)$$

The conservation of momentum equation can be written:

$$m_1 v_0 = m_1 v_1 + m_2 v_2 \quad \text{or:} \quad v_0 = v_1 + v_2 \qquad (8)$$

By combining (8), (7), and (6), we get the simple result:

$$R = \frac{1}{2}\left(1 + r^2\right) \qquad (9)$$

which shows that the final kinetic energy is the same as the initial only if the collision is perfectly elastic (r = 1).

A similar expression may be derived for the general case of unequal masses. It shows that:

$$R = \frac{m_1 + r^2 m_2}{m_1 + m_2} \tag{10}$$

which proves again that kinetic energy is conserved when r = 1.

1.2 Experiment

1.2.1 Equal masses

Choose two identical gliders with bumpers. Place one glider at rest at about the center of the track and direct the other toward it with a speed of ~20cm/s. Use the photogates on GATE function to measure the speeds of the gliders (before and after the collision). In this arrangement, the collision is to a good approximation, an elastic one.
To arrange an inelastic collision, attach a small piece of Scotch tape or Velcro to each bumper spring.
In both these situations, determine the initial and final velocities, check the conservation of momentum and calculate the coefficient of restitution. Is kinetic energy conserved? What do you think it happens to the energy that is lost?

1.2.2 Unequal masses

Choose two unequal gliders and weigh them. Make qualitative observations for both $m_1 > m_2$ and $m_1 < m_2$. Choose one particular combination for measurements. Note that in general it is necessary to measure three velocities: v_1 and v_2 can be measured with the two photogates; v_0 with a stopwatch.
Investigate: conservation of momentum and energy.
Optional: investigate an inelastic collision with unequal masses.

1.2.3 Action at a distance: the magnetic interaction force

Small ceramic magnets can be attached to the ends of the gliders, by using double sided Scotch tape. Be sure that the magnets are oriented so that they repel each other, do not let them strike each other; they are made from a brittle material and are easily broken.
Raise the track by a small angle and arrange the gliders at the two ends. The glider near the raised end will come to equilibrium at a position where the magnetic force just balances the force $mg\sin\alpha$ down the track. Carefully measure the distance between magnets. You may repeat the measurement for several track angles α (make sure to measure the elevation so that α can be calculated). Calculate the magnetic force at each position and plot a graph showing ***force*** as a function of ***distance.***

1.3 Questions on Part 1

i) Suppose we could place a small explosive charge on one of the bumpers, so that it detonates at the time of collision, pushing the two gliders apart. Will the momentum still be conserved? Explain. Will kinetic energy be conserved?

ii) What effect will the viscous friction of the supporting air layer have on your conclusions regarding conservation of momentum?

2. Dissipative forces

2.1 Viscous damping

2.1.1 Theoretical background

In this series of experiments, you will study dissipative (damping) forces which act to dissipate mechanical energy. Among them, there are: friction, magnetic damping forces, interaction forces in collisions which are not perfectly elastic, etc.

The principal source of frictions on the air track is the viscosity of the thin layer of air between the glider and the track.

The viscous friction force may be written as:

$$F = -\frac{\eta A v}{d} \qquad (11)$$

where η is a constant characteristic of the fluid, called viscosity, A is the surface area of the air layer and d is its thickness.

The negative sign indicates that the direction of F is always opposite to that of the velocity. Force is proportional to velocity:

$$F = -bv \qquad (12)$$

where the constant b depends on the properties of the air layer (viscosity, thickness, surface area).

If a glider moves on a level track in the absence of any other forces except viscous forces, the equations of motion are:

$$F = ma \quad \text{or} \quad -bv = m\frac{dv}{dt} \qquad (13)$$

showing that the rate of decrease of velocity is proportional to the velocity itself.

To determine the distance the glider travels before stopping, we express dv/dt in terms of dv/dx, using the chain rule for derivatives:

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v \qquad (14)$$

By substituting (13) in (14), we obtain:

$$\frac{dv}{dx} = -\frac{b}{m} \qquad (15)$$

which can be integrated to give:

$$v = v_0 - \frac{b}{m}x \qquad (16)$$

Equation (16) shows that a glider launched at an initial speed v_0 comes at rest (v = 0) after traveling a distance:

$$x = mv_0/b \qquad (17)$$

An interesting application of the dissipative forces is provided by the air track analogue to a **bouncing ball**. If the track is tilted to an angle α and a glider is released from rest at the top end, it will bounce at the bottom end but will not regain its original height. After a series of bounces, the glider eventually comes to rest at the bottom of the tilted track. Since the gravitational force is conservative, the loss of energy is due entirely to the work done against the frictional force. The work done by friction during the first descent from initial position x_0 is:

$$W = -\int_{x_0}^{0} F\,dx = -\int_{x_0}^{0} (-bv)\,dx \qquad (18)$$

Since the acceleration of the glider is approximately a = gsinα, the speed v at any position x is given by:

$$v^2 = 2ax = 2gx\sin\alpha \qquad (19)$$

substituting the expression for v into (18) and integrating, we obtain:

$$W = -\frac{2b(2a)^{1/2}(x_0)^{3/2}}{3} \qquad (20)$$

The work done on the return trip is approximately the same, so the total change in energy due to viscous friction is 2W.
If we take the potential energies at the ends of the first complete trip:

- initial: $(mg\sin\alpha)x_0$
- final: $(mg\sin\alpha)x_1$,

the corresponding ***loss of energy*** is: $mg\sin\alpha\,(x_1 - x_0) = mg\sin\alpha\Delta x$
By combining this result with Eq.(20) we obtain:

$$\Delta x = -\frac{b(2x_0)^{3/2}}{3m(a)^{1/2}} \qquad (21)$$

Thus if only the viscous forces are responsible for the energy loss, the change in height *after the first bounce* is proportional to the 3/2 power of the original height, and so on.

Energy can be lost also by collisions of bumpers to the end pieces of the track. The ratio of kinetic energies just after and just before the impact is r^2 (r is the coefficient of restitution - see definition in Eq. 7).

2.1.2 Experiments

The track has to be carefully leveled. Launch a glider, measure its initial velocity and the total distance it travels before it stops. For this measurement, the bumpers may be considered perfectly elastic. Determine the damping constant b using eq. (17).
Add weights to the glider to approximately double its mass and repeat the above observations and calculations. How does the value of b change? Explain.

For the "bouncing ball" experiment, tilt the track about 5 milliradians. Release a glider from the top; record its initial position and its maximum height after each bounce. Note that the position at the bottom of the track may not be at the zero point of the scale, in which case it must be subtracted from each reading.
To analyze the bouncing ball data, it is useful to take the logarithm of both sides in Eq. (21):

$$\log(-\Delta x) = \frac{3}{2}\log x_0 + \log \frac{b\,2^{3/2}}{3m(a)^{1/2}} \qquad (22)$$

Thus, if the bouncing ball behaves according to this equation, the graph of $\log(-\Delta x)$ versus $\log x_0$ should be a straight line with a slope of 3/2. If not only the viscous forces, but also the collision to the end of the track bumper determines the energy loss, the slope of the graph should be between 1 and 3/2.

2.2 Magnetic damping

2.2.1 Theoretical background

When an electrical conductor moves through a magnetic field, the changing magnetic flux in the conductor induces currents of magnitude proportional to the rate of change of flux and thus to the velocity. These *eddy currents* in turn experience a force which at each point is proportional to the field at that point.
The direction of the force on the conductor is always such as to oppose the relative motion:

$$F = -b'v \qquad (18)$$

In this case, the constant b' is proportional to the electric conductivity of the material, to the area of the conductor over which the magnetic field extends and to the square of the magnetic field intensity.

2.2.2 Experiment

Attach four magnets symmetrically to the glider. Attach enough weight to another glider to give it the same total mass as the glider with magnets. Place the two on the track and push them together (the magnetic glider in back) to give them the same initial velocity. Note that the magnetically damped glider lags increasingly behind the other. Determine the total damping constant for the magnetically damped glider by the same method described above. Note that you are measuring the 'total b' due to both viscous and magnetic damping.

2.3 Questions on Part 2

i) For a glider with only viscous air damping, how does the damping constant b vary with the mass of the glider? Why should this variation be expected?

ii) When a glider on a tilted track is given an initial velocity v_0, show that if the track is sufficiently long, the glider will reach a final velocity (terminal velocity) which is independent of v_0. Derive an expression for the terminal velocity.

iii) Is the effect of air surrounding the glider significant in comparison with the effect of the air layer between glider and track, in determining the total frictional force? Explain.

Part 3 Periodic motion

3.1 Theoretical background

When the force on a body is proportional to the displacement of the body from equilibrium and is directed toward the equilibrium position, there is a repetitive back-and-forth motion about this position, called *periodic motion*.

The oscillations of a mass on a spring, the motion of a pendulum, the vibrations of a stringed musical instrument, are familiar examples of periodic motion.

Let the displacement of the mass m from equilibrium be x. The force is given by:

$$F = -kx \qquad (19)$$

where k is a constant called the *force constant* for the system.

According to Newton's second law:

$$-kx = ma = m\frac{d^2x}{dt^2} \qquad (20)$$

It is easy to verify that functions:

$$\begin{aligned} x &= x_0 \cos\omega t \\ x &= x_0 \sin\omega t \end{aligned} \qquad (21)$$

are solutions of equation (21), where x_0 is a constant called the amplitude and ω (the angular frequency) is defined by:

$$\omega = \sqrt{\frac{k}{m}} \qquad (22)$$

Each time the quantity ωt increases by 2π, the motion goes through one cycle. The time for one cycle is called period (T):

$$T = \frac{2\pi}{\omega} = 2\pi\left(\frac{m}{k}\right)^{\frac{1}{2}} \qquad (23)$$

The number of cycles per unit time is called frequency (f):

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$f = \frac{1}{T} \qquad (24)$$

Figure 1 presents a model, comprising of a glider placed on a horizontal air track and attached at its ends to identical springs.

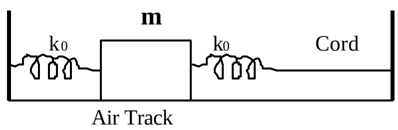

Figure 1

The force constants of the two springs are equal to k_0. In order to stretch either spring by x, a force equal to k_0x has to be applied.
The mass m is displaced by distance x to the right of its equilibrium position. The force of the left spring increases by k_0x while that of the right one decreases by the same amount. The result is a net force to the left with magnitude $2k_0x$, so the force constant to be used in Eq. (19) and (20) is $k = 2k_0$.
The total energy of the system from Fig. 1 is conserved. When the mass reaches the endpoints of its motion and stops, the energy is entirely potential energy; when it passes the equilibrium position, the energy is entirely kinetic energy. The average potential energy is equal to the average kinetic energy and each is equal to half the total energy.
The presence of the damping forces in a real system determines a progressive decrease in the amplitude of the oscillations. The position of the mass is given by a more complicated function than (21).
Following the experiments from Part 2, we assume that the damping force is given by:

$$F = -bv = -b\frac{dx}{dt} \qquad (25)$$

where b is the damping constant, characterizing the strength of the damping force.
The rate at which oscillations die away depends on the magnitude of b; a large value of b means rapid decay, and the converse.
By incorporating (25) is Newton's second law, we obtain:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \qquad (26)$$

For the purpose of our experiment, we shall use an approximate analysis approach of the damped oscillations, based on energy considerations.
The energy lost during one cycle is given by the average rate of energy loss, calculated over that cycle.
The rate of energy loss is given by $(bv)v = bv^2$.
To find the average value of v^2, we remember that the average kinetic energy for a harmonic oscillator is equal to its average potential energy and each of these quantities equals half the total energy E:

$$\frac{1}{2}m\left\langle v^2 \right\rangle_{av} = \frac{1}{2}E \qquad (27)$$

The average rate of loss of energy is then:

$$\left\langle \frac{dE}{dt} \right\rangle_{av} = -\left\langle bv^2 \right\rangle_{av} = -\frac{b}{m}E \qquad (28)$$

The time required for one cycle is given by Eq. (23).
The energy loss during one cycle is:

$$\Delta E = -\left(\frac{b}{m}E\right)\left(\frac{2\pi}{\omega}\right) = -2\pi\frac{b}{(km)^{1/2}}E \qquad (29)$$

Eq. (28) is a differential equation for E; its solution gives the energy as a function of time:

$$E = E_0 e^{-(b/m)t} \qquad (30)$$

where E_0 is the initial total energy at time t = 0.
The time required for the energy to decrease to 1/e of its initial value is called *relaxation time* and is given by (m/b).
A useful constant that can be calculated now is called *quality factor Q*. It is defined as:

$$Q = \frac{2\pi E}{\Delta E} = \frac{m\omega}{b} = \frac{(mk)^{1/2}}{B} \qquad (31)$$

which means 2π times the ratio of maximum energy stored in the system to the energy dissipated in one cycle.
The amplitude is the most directly observable variable in an experiment on periodic motion. It decreases with time as:

$$x = x_0\left(e^{-(b/m)t}\right)^{1/2} \qquad (32)$$

The exponential function above is the square root of the function describing the time variation of E (E is proportional to x^2).
The *relaxation time* for the amplitude of the oscillations can now be expressed as:

$$\tau = \frac{2m}{b} \qquad (33)$$

The *half-life* $T_{1/2}$ is defined as the time during which the amplitude drops to half its original value:

$$T_{1/2} = \tau \ln 2 = \frac{2m \ln 2}{b} = \frac{1.386m}{b} \qquad (34)$$

Eq. 33 can be used to give the quality factor Q:

$$Q = \frac{1}{2}\omega\tau = \frac{1}{2}\left(\frac{2\pi}{T}\right)\left(\frac{T_{1/2}}{\ln 2}\right) = \frac{\pi}{\ln 2}\frac{T_{1/2}}{T} \qquad (35)$$

3.2 Experiment

3.2.1 Spring constant

In order to compare the theoretical predictions with the observed behavior of the system, the mass and spring constants must be known.

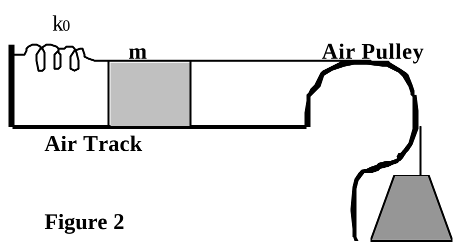

Figure 2

A suggested procedure for measuring the spring constant is shown in Fig.2. The spring is attached between the end of the track and a glider. Use a piece of magnetic recording tape (get it from the Resource Center), tie it to the glider, pass it over the air pulley and attach a weight to the other end. Note the equilibrium position of the reference line on the glider. Add weights in 10 g increments, up to 100 g, recording the position of the reference line on the glider for each weight. Do not stretch the spring more than 20 cm; beyond this it will be permanently deformed.
Plot extension of spring as a function of applied force (weight). From the graph, determine the constant k_0.

3.2.2 Simple harmonic motion.

To observe simple harmonic motion, remove the tape and attach a second spring as in Fig. 1. Attach a piece of cord to the end of this spring. Pull the cord enough to stretch each spring about 10 cm and tie it to the end of the track. Displace the glider ~5 cm from its equilibrium position and release it.
Time 10 cycles of the motion; find the period and the frequency. Repeat the measurement with smaller and larger vibration amplitudes. Record the amplitude for each trial. Is there any significant variation in frequency if larger or smaller amplitudes are used?
From the measured frequency and force constants, calculate the mass of the glider.

3.2.3 Damping

Displace the glider 5 cm from the equilibrium and release it with no initial velocity. Count the number of cycles for the amplitude to decrease to half its original value. Calculate the quality factor of the system, Q, and also the relaxation time τ. Also compute the damping constant b, compare with the value obtained in Part 2.
Add damping magnets to the glider and again determine Q. Compare the result with q for a glider of the same mass but only viscous damping.
Add mass (slotted weights) to the glider and observe how Q changes with mass. Can you explain why Q varies?

3.2.4 Coupled oscillators

Assemble the system as shown in Fig. 3:

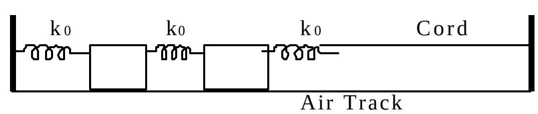

Figure 3

Pull the cord tight enough to extend each spring by ~10 cm. Displace one mass, holding the other fixed and release both masses at once. Observe the complex nature of the motion.
Displace both masses toward the center by the same amount and release them. Does the motion appear to be sinusoidal? This mode in which the masses move in opposite directions is called the *symmetric mode*.
Measure the time for 10 oscillations, compute the period or frequency and compare with:

$$\omega = \left(\frac{3k_0}{m}\right)^{1/2} \qquad \text{(see [2], p.23)}$$

Displace the two masses in the same direction by equal amounts and release them. Is the motion sinusoidal? Determine again the frequency and compare with the theoretical prediction. This mode is called *antisymmetric*.

3.3 Questions to Part 3

i) How is the motion of the system shown in Fig. 1 related to the motion of a simple pendulum?

ii) If two identical springs, each with spring constant k_0, are connected in series, what is the resulting spring constant? What if they are connected in parallel?

iii) In the above analysis of harmonic oscillators, the masses of the springs have been neglected. Will the effect of spring mass be to increase or decrease the frequency? Explain.

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