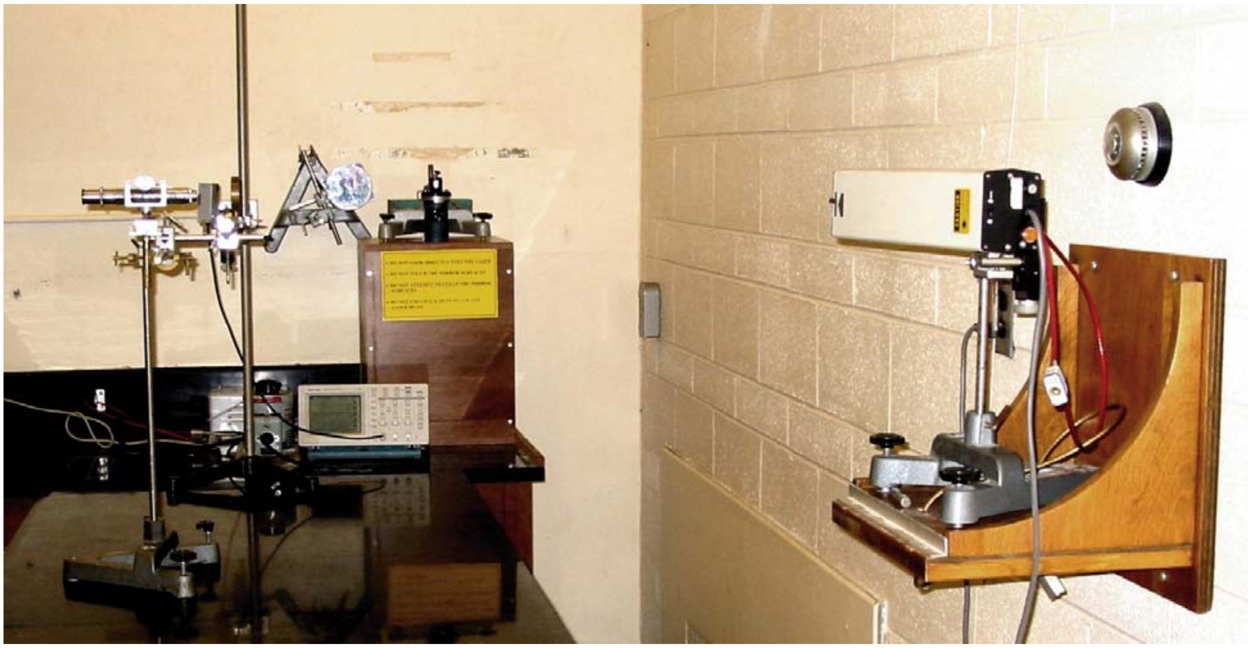
THE SPEED OF LIGHT



CAUTION:

- **Never look directly into the beam without a diffusing screen between you and the laser. If viewed directly, the beam from even a low power laser can cause permanent damage to your vision.**
- **Do not touch the surfaces of the mirrors. The mirrors are aluminized-front-surface mirrors and do NOT have a protective overcoating. Touching, or any attempt to clean the mirror surfaces will result in damage to the mirrors.**

INTRODUCTION

This experiment is a modification of Foucault's method for the measurement of the speed of light using a rotating mirror. In 1862, Foucault measured **c** = (298 000 ± 500) km/s, or an accuracy of ± 0.17% using a baseline of only 20 m. In 1929 Michelson used a development of the rotating mirror arrangement over a baseline of 22 miles between Mt. Wilson and Mt. San Antonio in California to determine **c** = (299 796 ± 4) km/s. Although the technique you use is similar, you can probably only expect to achieve accuracies of ± 5%. You will find as did Foucault, that there is a limit to the accuracy achievable in a technique dependent on the measurement of the displacement of a light beam. It is interesting, however, considering the magnitude of the velocity being measured, that one can quite easily measure **c** in one or two laboratory periods.

The other challenge of this experiment is the solving of the problem of how to set-up an apparatus systematically. If you work out your system of alignment carefully, you can perform the experiment easily. However, if you do not proceed systematically, you could spend a couple of lab periods without obtaining any results.

THEORY

A schematic diagram of the apparatus is given in Figures 1 and 2.

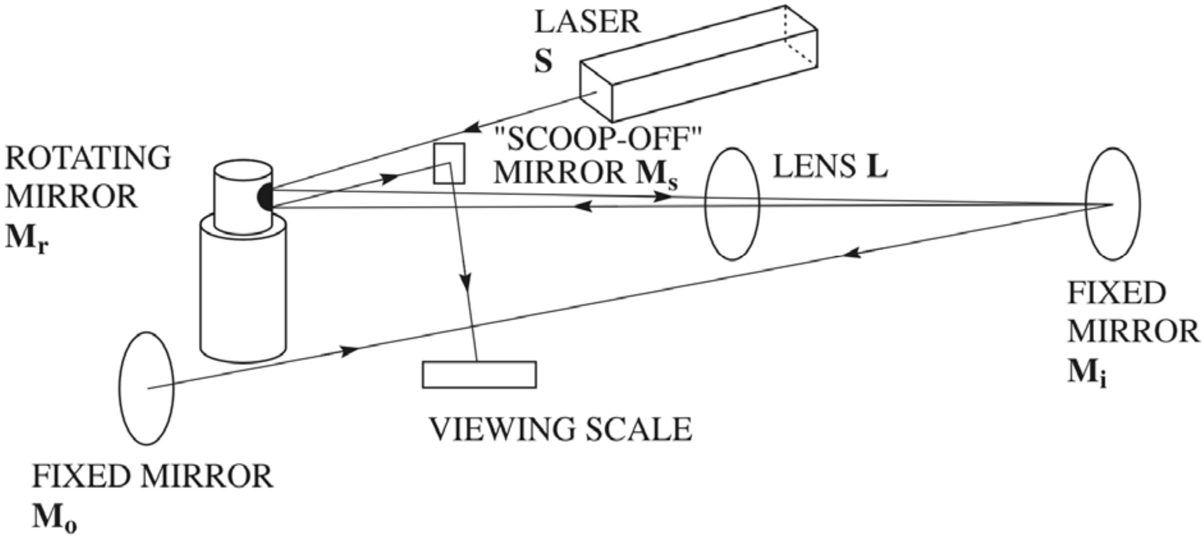


Figure 1

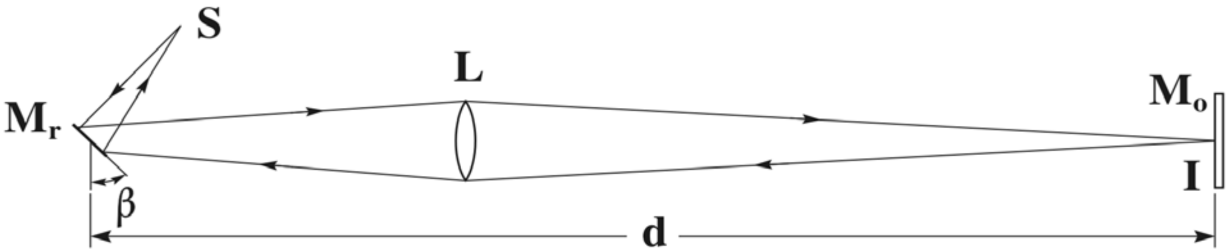


Figure 2

Light from a source **S** falls onto the rotating mirror $\mathbf{M_r}$ and is focussed by the lens **L** to form an image **I** on the surface of the mirror $\mathbf{M_o}$. The purpose of the intermediate mirror $\mathbf{M_i}$ (not shown in Figure 2) is to create a longer path by allowing the light to double back in the room. The light from the image on $\mathbf{M_o}$ will be reflected and form a second image exactly coincident with the original object **S** since **I** and **S** are conjugate points of the lens **L**. As a result of the focussing properties of **L**, all the rays that pass through **L** and reach $\mathbf{M_o}$ will reflect and form and image coincident with the object **S** even though the rays will not generally retrace their paths exactly.

An angle β is defined as the angle between the plane of the mirror and some arbitrary direction as shown in Figure 2. It is important to note that the position of the final image on return to the source location **S** is independent of the angle β of the rotating mirror $\mathbf{M_r}$, although the position of the image **I** on the face of mirror $\mathbf{M_o}$ will vary with angle β. Of course, this is true only if the rays do pass through lens **L** and strike the surface of $\mathbf{M_o}$. If they don't, the light just won't make it back to the location of **S**.

What is described above is true when $\mathbf{M_r}$ is stationary at any reasonable angle β or rotating at some low value of angular velocity $\omega = \frac{d\beta}{dt}$. However, if ω is sufficiently large so that the mirror $\mathbf{M_r}$ has rotated through an angle δβ in the time the light takes to travel from $\mathbf{M_r}$ to $\mathbf{M_o}$ and back, the final image will be displaced sideways from the original source position **S**.

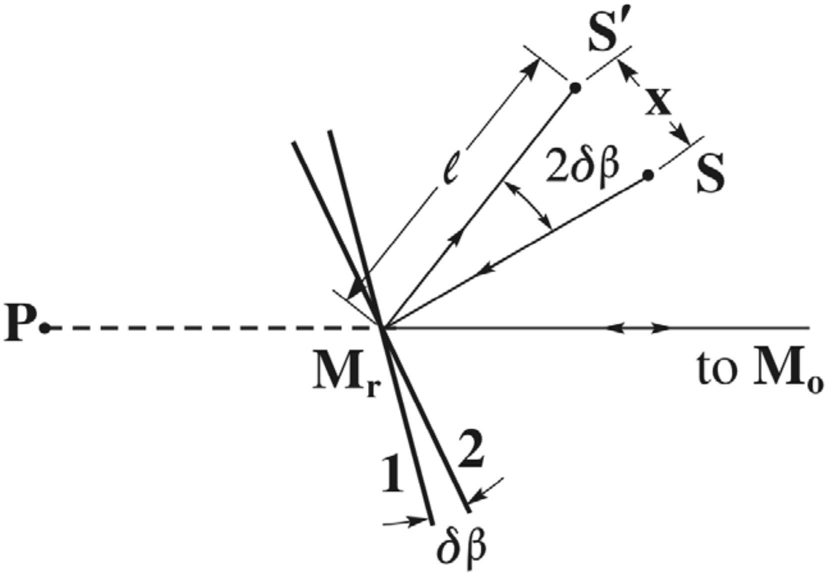


Figure 3

Referring to Figure 3, if the rotating mirror is initially at position **1** then a line from **S** to **P** is perpendicular to $\mathbf{M_r}$ and the source **S** produces an image in $\mathbf{M_r}$ at **P**. The light, after reflection from $\mathbf{M_o}$ heads to **P** on its return and forms an image at **S**. But if $\mathbf{M_r}$ is now at position **2**, the light is reflected at a larger angle to form an image at **S'**. If the mirror has rotated through an angle δβ, using the laws of reflection, you should be able to show that the angle subtended at the mirror $\mathbf{M_r}$ by **S** and **S'** is 2 x δβ. Thus, the displacement **x** of the final image, due to change in mirror angle from when the light leaves $\mathbf{M_r}$ to when it returns is just $\mathbf{x} = \ell \times 2\delta\beta$, assuming δβ to be small, and where ℓ is the distance from $\mathbf{M_r}$ to **S**.

If the optical path length between $\mathbf{M_r}$ and $\mathbf{M_o}$ is **d**, then the light, travelling at a speed **c**, takes a time $\delta t = \frac{2\mathbf{d}}{\mathbf{c}}$ between reflections at $\mathbf{M_r}$, so that if the mirror is rotating at an angular velocity ω, then $\delta\beta = \omega \times \delta t = 2\omega\frac{\mathbf{d}}{\mathbf{c}}$ and thus

$$\mathbf{x} = \frac{4\ell\mathbf{d}}{\mathbf{c}}\omega \tag{1}$$

The determination of **c** thus reduces to the measurement of two fixed distances, a variable angular velocity, and the displacement of an imaged light beam.

THE EXPERIMENT

Although the method requires merely a small intense light source we have provided you with a laser light source. Lasers have the advantage of lots of intensity, all in the direction you want.

SETUP

The rotating mirror $\mathbf{M_r}$ and the fixed mirrors $\mathbf{M_o}$ and $\mathbf{M_i}$ are placed to give the maximum path length possible given the finite size of the space. The rotating mirror $\mathbf{M_r}$ can be rotated by hand by inserting the removable knob into the top of the motor assembly. As a safety feature, the removable knob is tethered to an elastic cord. Removing the knob before starting the motor prevents accidental damage to the motor shaft.

Preliminary Beam Alignment

With lens **L** removed from the light path, use the following steps as a guide to form the path

$$\mathbf{S} \Rightarrow \mathbf{M_r} \Rightarrow \mathbf{M_i} \Rightarrow \mathbf{M_o} \Rightarrow \mathbf{M_i} \Rightarrow \mathbf{M_r} \Rightarrow \mathbf{S}$$

1. Adjust the laser **S** and its stand so that the beam strikes the centre of $\mathbf{M_r}$ and reflects back to **S**.

2. Using the cement blocks in the wall of the room as a guide, adjust $\mathbf{M_r}$ and the laser so that the beam rotates in a horizontal plane as $\mathbf{M_r}$ is rotated. When making corrections to the alignment, make only partial corrections with either $\mathbf{M_r}$ or **S**. Note that when $\mathbf{M_r}$ is adjusted, **S** must also be simultaneously adjusted so that the beam still strikes the centre of $\mathbf{M_r}$ and still reflects back to **S**. This procedure may require several iterations.

3. Check that the centre of $\mathbf{M_o}$ is at the same height as the centre of $\mathbf{M_r}$. If it is not, make the necessary adjustments.

4. Adjust $\mathbf{M_i}$ so that the beam hits its centre and use the adjustment screws so that the reflected beam strikes the centre of $\mathbf{M_o}$.

5. Adjust $\mathbf{M_o}$ so that the reflected beam hits the centre of $\mathbf{M_i}$ and, even though it is large, the centre of the beam reflecting from $\mathbf{M_i}$ again strikes $\mathbf{M_r}$. You should see a large red spot which is a rectangular image of $\mathbf{M_r}$ back at the laser source **S**.

Introduction of the Lens L

6. Calculate the position for lens **L** to make **S** and $\mathbf{M_o}$ optically conjugate (i.e., **L** causes an object at **S** to form an image at $\mathbf{M_o}$). Use the thin lens formula $\frac{1}{\mathbf{s}} + \frac{1}{\mathbf{s'}} = \frac{1}{\mathbf{f}}$ where $\mathbf{s}$ and $\mathbf{s'}$ are the object and image distances respectively. You will need to know the total distance from **S** to $\mathbf{M_o}$. The focal length **f**, of **L**, is given in millimetres on the lens holder. Although the correct answer requires the solution of a quadratic equation, you should be able to estimate the correct answer in your head.

7. Place **L** in its calculated location and adjust its vertical and transverse position so that the beam now traverses the path

$$\mathbf{S} \Rightarrow \mathbf{M_r} \Rightarrow \mathbf{L} \Rightarrow \mathbf{M_i} \Rightarrow \mathbf{M_o} \Rightarrow \mathbf{M_i} \Rightarrow \mathbf{L} \Rightarrow \mathbf{M_r} \Rightarrow \mathbf{S}$$

Introduction of the “Scoop Off” Mirror $\mathbf{M_s}$

By this point in the set-up, you have the beam returning to the laser source **S**. If you were to now run the motor to rotate $\mathbf{M_r}$ you would not be able to make measurements since this would require that your eye be inside the laser. Thus you must now insert the "scoop-off" mirror $\mathbf{M_s}$ between the laser **S** and $\mathbf{M_r}$, angled in such a way that the returning light beam gets reflected on to your observing scale rather than returning to the laser **S**.

Since the beam is very intense, use the coloured filter provided to reduce the brightness when viewing the beam through the magnifying eyepiece. Note that the smallest divisions on the viewing scale are 0.5 mm. The coloured filter is to be removed later when $\mathbf{M_r}$ is in motion.

7. The "scoop-off" is achieved by first making a very slight adjustment of one foot of the laser stand so that the beam strikes $\mathbf{M_r}$ near its top. Blocking $\mathbf{M_o}$ will make this adjustment easier.

8. Probably the returning beam will be below $\mathbf{M_r}$ (there may be multiple reflected beams but the returning beam will be the brightest). **M**ake a very slight adjustment of the tilt of $\mathbf{M_o}$ so that the returning beam strikes $\mathbf{M_r}$ near its bottom. Use a piece of paper near $\mathbf{M_r}$ to view the separation between the outgoing and returning beams.

9. Now $\mathbf{M_s}$ may be placed near $\mathbf{M_r}$ such that it intercepts only the beam travelling from $\mathbf{M_r}$ headed towards **S,** but does not intercept any of the three other light beams arriving at or leaving $\mathbf{M_r}$. Adjust the angle of $\mathbf{M_s}$ so that the outgoing beam hits the viewing scale.

10. Position the viewing scale at such a distance that the light spot is a minimum size. Although you should not move $\mathbf{M_r}$ now, it should be pointed out that at this position the spot will not move from side to side when $\mathbf{M_r}$ is rotated through small angles. Later when $\mathbf{M_r}$ is rotating, the beam hits the scale for only a small range of angles and this is why you see a circular spot and not a flattened ellipse.

11. Turn on the autotransformer and set the motor rotating at a low speed. Remove the coloured filter from the beam path. Using the eyepiece, fine tune the position of the scale so that the spot on the scale is round and small in size.

Measurements

12. Note that the definition of ℓ now becomes the distance from $\mathbf{M_r}$ to $\mathbf{M_s}$ to the viewing scale. When the viewing scale is placed at the position where the spot size is a minimum, theory says that the light path distance from $\mathbf{M_r}$ to $\mathbf{M_s}$ to the viewing scale is the same as the distance from **S** to $\mathbf{M_r}$. Measure ℓ.

13. Measure **d**.

14. The time of rotation of the mirror and hence ω can be measured using the photodiode. Connect the photodiode to **CH1** of the oscilloscope. Press **CH1 Menu** and set:
    Coupling - DC
    BW Limit - ON
    VOLTS/DIV - Coarse
    Probe – 1X
    INVERT - OFF

    With $\mathbf{M_r}$ stationary and with the detector in a position such that no part of the beam is intercepted by the motor housing, adjust the photodiode for maximum signal which should be greater than 5 volts.

15. To determine the frequency when $\mathbf{M_r}$ is rotating, press **MEASURE**, choose **Type** and set **CH1** to **Freq**. Adjust **VOLT/DIV**, **SEC/DIV** and **TRIGGER LEVEL** as the motor speed changes to keep at least two peaks on the screen and the frequency measurement valid.

    Without touching the mirror surfaces, when you determine ω, be sure to check whether the rotating mirror is single or double sided as this will affect your calculations.

16. Measure **x** as a function of ω over as large a range of ω as is practical. Take some of your data with ω increasing and some with ω decreasing and keep track of the order in which data is recorded. At high frequencies the photodiode may have to be adjusted for maximum signal.

DATA INTERPRETATION

The form of equation (1) suggests a graphical way of interpreting the data. The **x** in equation (1) represents the displacement of the light spot from its position when the mirror is not rotating. The measuring scale will probably be arbitrarily positioned with its zero not corresponding to the position of the spot for a stationary $\mathbf{M_r}$. Thus equation (1) is better written as:

$$\mathbf{x} = \frac{4\ell\mathbf{d}}{\mathbf{c}}\omega + \mathbf{x_o} \tag{2}$$

Do not try to determine $\mathbf{x_o}$ with the mirror stationary as the brightness of the spot is so much greater than when the mirror is rotating that the data taken under the two intensity conditions are not comparable.

SUPPLEMENT

One may be able to increase the precision of the measurements by inserting a slit directly in front of the laser. The slit will now be the source. Choose the slit width and slit position to optimize clarity, sharpness and intensity of image as seen in the viewing eyepiece. Note that it will be more difficult to measure higher frequencies on the oscilloscope since the intensity of the light striking the photodiode will now be lower.

(jv – 1988, jp - 2006)