Radioactive Half-Life

Introduction

The tendency of a radioactive nucleus to decay can be specified by the *half-life*. Given a large number of identical radioactive nuclei, in one half-life 1/2 of them will decay. If we wait another half-life, 1/2 of the remaining nuclei will decay. Put another way, in one half-life the *probability* that a nucleus will decay is 50%.

You may wish to muse about the fact that above we said we have *identical* nuclei, but in one half-life 1/2 of the nuclei will decay and the other 1/2 will not. If they nuclei really were identical, then whether or not one of them decays in one half-life is pure chance. This view is central to *Quantum Mechanics*. Einstein never believed in Quantum Mechanics, and this probabilistic nature of the quantum world view is one reason. As he said repeatedly, "God does not play dice with the universe." Bohr, who did believe in Quantum Mechanics, responded, "Quit telling God what to do!"

In this Module you will simulate radioactive decays by throwing dice, and investigate the half-life and the related *decay constant*.

Duration

This Module should take about two hours to complete.

Preparation

You should look over this Guide to the Module before coming to your Practical. You may also wish to look over the sections of your textbook that discuss radioactivity.

Some Background Information

If we have **N** identical radioactive nuclei, then their rate of decay is:

$$\frac{dN}{dt} \tag{1}$$

Note that this is a negative number, since the number of nuclei is decreasing in time. The rate of decay is proportional to the number of nuclei: if we have more nuclei then more of them will decay in a given period of time. Thus:

$$\frac{dN}{dt} = -\lambda N \qquad (2)$$

λ is a constant for a particular type of nucleus, and is called the *decay constant*.

It is fairly simple to solve Eqn. 2 to get:

$$N = N_0 e^{-\lambda t} \qquad (3)$$

where $\mathbf{N_0}$ is the number of nuclei at time 0.

When the time is equal to one half-life, $\mathbf{T_{1/2}}$, then **N** = 1/2 $\mathbf{N_0}$.

$$\frac{1}{2} N_0 = N_0 e^{-\lambda T_{1/2}} \qquad (4)$$

Taking the natural logarithm of both sides and doing a small amount of algebra gives:

$$T_{1/2} = \frac{-\ln\left(\frac{1}{2}\right)}{\lambda} \simeq \frac{0.693}{\lambda} \qquad (5)$$

You may see a Flash animation of nuclear decay by clicking on the blue button to the right.

Another Notation

We have been discussing the half-life in terms of the decay constant λ. You may wish to know that there is another notation in common use. Instead of Eqn 3:

$$N = N_0 e^{-\lambda t}$$

we can write:

$$N = N_0 e^{-t/\tau} \qquad (6)$$

Here the tendency of a nucleus to decay is given by the *lifetime* τ instead of the decay constant λ . The relation between these 2 ways of describing radioactive decays is:

$$\lambda = \frac{1}{\tau} \tag{7}$$

The Practical

You have 100 dice.

Question 1: If you throw one of the dies onto the supplied tray, what is the probability that it will end up with the six facing up?

Question 2: Above we mentioned that according to Quantum Mechanics, whether or not a particular nucleus decays is truly random. Is the throwing of the die similarly random?

Part 1

Place all 100 dice into the supplied jar. Call this starting point *Trial 0*, and clearly the initial number of dice $\mathbf{N_0}$ = 100.

Shake the jar, and pour the dice onto the tray. Call this *Trial 1*. Let us assume that when a die has a six facing up, that this corresponds to it having decayed since the starting point.

Count and record the number of dice that came up six. Remove those dice and record how many are remaining. Using the notation introduced above, this is **N**.

Place the remaining dice into the jar, shake, and pour them onto the tray. This is *Trial 2*. Again count and record the number of dice with a six, remove them, and record how many are remaining

Repeat this process until only a few dice are left.

On the whiteboard sketch the number of remaining dice **N** versus the trial number.

It is likely that your data will be close to but not exactly equal to what would be expected from pure probability. On the whiteboard calculate what the number of remaining cubes versus trial number should theoretically be, and add those points to your sketch. Devise a method (color, shape) that clearly distinguishes between the experimental results and the theoretical prediction.

Question 3 : Say for Trial **m** you experimentally measured N_{expt} remaining dice but have calculated a theoretical value N_{theory}. Which of these 2 values for **N** should you use to calculate the theoretical value after the next Trial?

Question 4 : After what number of trials would you expect exactly one-half of the dice to be remaining? If your value is not an integer, does this indicate a mistake?

Question 5 : Imagine that the trials were done every 60 seconds. What is the half-life of the dice in seconds?

Question 6 : For trials done every 60 seconds, what is the *decay constant* λ in s^{-1}?

Part Two

Put all 100 dice into the jar, shake, and pour onto the table. Record the number that have "decayed."

Repeat for all 100 dice a few times, recording the number that have "decayed" each time.

On the whiteboard sketch a histogram of the number of decays.

Question 7 : If you repeated this measurement many times, what would you expect the shape of the histogram to be?

Question 8 : What would you expect the *standard deviation* of the distribution to be?

Part Three

This Part does not have an associated activity.

Imagine that we decide that if a die comes up either **six** or **one** that corresponds to it having decayed.

Question 9 : What is the probability that one of the die will have "decayed" after the first trial?

Questions 10 : What is the half-life in seconds of the dice for this case?

Question 11: What is the *decay constant* λ in s^{-1} for this case?

Equipment

- 100 large dice.
- A jar large enough to contain all 100 dice.
- A tray large enough to contain all 100 dice.

This module was written by David M. Harrison in March 2005.
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This Module is based on one developed at Sydney University, Australia