Racing Cylinders: Instructor Notes

This document contains notes for the Instructor on the Racing Cylinders Module. The contents should not be made available to students.

You should be aware that most students discussed linear kinematics and dynamics in high school. The topic of rotational motion, which this Module deals with, is discussed hardly at all, and is conceptually difficult for many students. Thus this topic is when the "rubber hits the road" in this course.

Preparation

Be sure to look over the Guide to the Module before the Practical. If you have not done anything with the moment of inertia in a while you may wish to look at the textbook to refresh your memory.

Measuring the Size and Mass of the Cylinders

Your students should measure the radii with the vernier caliper, which is much more precise and accurate than, say, using a ruler. Remember that they will not need to know the lengths of the 2 cylinders, although they are the same.

Question Answers

■ Question 1

The "key" here is that if the mass is distributed a greater distance away from the axis of rotation then the moment of inertia is greater. In fact, it scales as the *square* of the distance from the axis of rotation.

■ Question 2

For a given speed at the bottom of the plane, the total kinetic energy is distributed between the translational part and the rotational part:

$$\frac{1}{2} M v_f^2 + \frac{1}{2} I \omega_f^2$$

The total kinetic energy is just the change in potential energy, so is the same for the 2 cylinders. But for the hollow cylinder **I** is larger, so more of the kinetic energy is taken up by the rotational part. Thus, it will be moving more slowly than the solid cylinder.

It is probably quite acceptable for your students to get this far and just say that the cylinder with the greatest final speed is the one that reaches the bottom first. Thus the answer is that **the solid cylinder reaches the bottom first**.

However, here is the kinematics that proves the relationship between final speed and time to get to the bottom of the ramp.For both cylinders the acceleration is constant, with different values for each. Say the length of the plane from where the cylinders were released to the bottom is **L**. Then:

$$L = \frac{1}{2} a t^2$$

But we also know that for constant acceleration with zero inital speed:

$$a = \frac{v_f}{t}$$

Thus:

$$L = \frac{1}{2} \frac{v_f}{t} t^2 = \frac{1}{2} v_f t$$

So:

$$t = \frac{2L}{v_f}$$

So the greater final speed is the smaller time to get to the bottom.

■ Question 3

One way to look at this is to just realise that the cylinder is rotating with angular speed ω about the point in contact with the plane. So the speed of the axis of rotation is just:

$$v_{axis} = \omega R$$

Similarly, the point at the top of the cylinder will have a speed:

$$v_{top} = \omega 2 R$$

There is another way to think about this. Think about looking at the rolling cylinder in a frame of reference where the axis is stationary. Then the plane is seen to be moving with a speed**:**

$$-v_{axis}$$

This is also the speed of the point of the cylinder in contact with the plane, and since in this frame that point is just in uniform circular motion the usual simple relation applies:

$$- v_{axis} = - \omega R$$

■ Question 4

The work done is the final kinetic energy:

$$m g h = \frac{1}{2} M v_f^2 + \frac{1}{2} I \omega_f^2 = \frac{1}{2} M v_f^2 + \frac{1}{2} I \frac{v_f^2}{R^2}$$

Solving for **I**:

$$I = M R^2 \left(\frac{2 g h}{v_f^2} - 1\right)$$

Possible Further Topics for Discussion

Here are some things that you may wish to lead a Learning Team to think about.

■ 1. Generic Moment of Inertia

For any object at all, the moment of inertia can be written as:

$$I = f M R^2$$

where **f** is a number, **M** is the total mass, and **R** is some dimension of the object about its axis of rotation.

■ 2. Where Does Equation 2 Come From?

One answer uses integrals in a fashion which is beyond most of our students at this point in the year.

$$I = \int_{R_i}^{R_o} r^2 \, dm$$

If the density of the cylinder is ρ and the length of the cylinder is **L**, then:

$$dm = L \rho \, dr \, r \, d\theta$$

so:

$$I = \int_{R_i}^{R_o} \int_0^{2\pi} r^3 L \rho \, d\theta \, dr$$

The total mass **M** is:

$$M = \rho (\pi R_o^2 - \pi R_i^2) L$$

These can be solved to get Eqn. 2. However, there is a less mathematically sophisticated approach. We imagine 2 solid cylinders. One has a radius equal to the outer radius of the hollow cylinder, and the other has a radius equal to the inner radius of the hollow cylinder. Then the moment of inertia of the hollow cylinder is the difference between the moments of inertia of these two:

$$I = \frac{1}{2} M_1 R_o^2 - \frac{1}{2} M_2 R_i^2$$

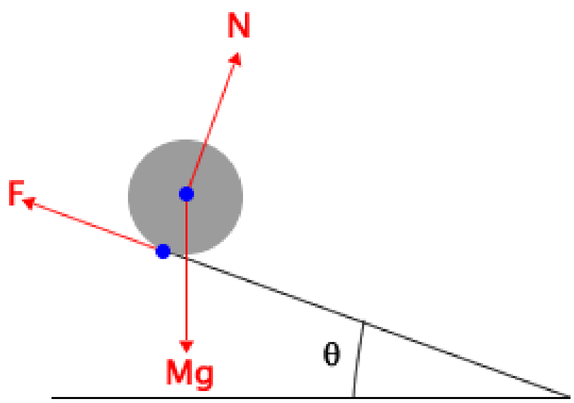
The mass of the hollow cylinder is:

$$M = M_1 - M_2$$

It is a bit laborious but not too difficult to derive Eqn. 2 from these 2 relations.

■ 3. Why use Energy for Questions 2 and 4?

I don't think that having the students actually doing this analysis using torques, forces, etc. is worthwhile. But having them begin to set it up can quickly show them that it really is the hard way to do it. First, here is the free body diagram.



N is the normal force exerted on the cylinder by the plane, **mg** is the cylinder's weight, and **F** is the frictional force exerted on the cylinder by the plane. Since the contact point between the cylinder and the plane is stationary, we will analyse about that point. Now we need to know that the moment of inertia about that point is related to the moment of inertia about the axis by an added term of $\mathbf{M}R^2$. This is called the *Parallel Axis Theorem*, which most of our first year courses do not discuss. Ugh!

For the curious and mathematically inclined, here is the way to do the solution. The relation between the torque and the angular acceleration is:

$$M g R \sin(\theta) = (I + M R^2)\,\alpha = (I + M R^2)\,\frac{a}{R}$$

The kinematical relation between the final speed and the acceleration if the cylinder rolls a distance **L** is:

$$v_f^2 = 2\,a\,L$$

The relation between the distance **L** and the vertical height **h** is:

$$h = L \sin(\theta)$$

These three equations can be solved for **I** to duplicate the answer to Question 4.

■ 4. Fractals

Most of our first year courses do not talk about chaos, fractals and similar topics, but the document on the Basal Metabolism linked to from the Guide briefly mentions fractals. If a Learning Team is interested in exploring a topic that is probably not examinable in their course, this is a fascinating one that they may wish to learn more about.

Author

These notes were written by David M. Harrison in April, 2005.