MOMENT OF INERTIA

INTRODUCTION

The property of a body by which it resists acceleration is called the inertial mass *m*. The rotational analogue to inertial mass is the moment of inertia *I* and it is the property of a body by which the body resists angular acceleration. Newton's second law of motion $\vec{F} = m\vec{a}$ for linear motion has a rotational analogue which is $\vec{\tau} = I\vec{\alpha}$ where $\vec{\tau}$ is the torque and $\vec{\alpha}$ is the angular acceleration.

In this experiment you will determine the moments of inertia of various objects by applying varying torques to a body and measuring the corresponding angular accelerations.

In this experiment, data is taken using rotational motion sensors. The rotational motion sensors are interfaced to a computer and the data is analysed using the DataStudio software.

THEORY

If a torque is applied to a body which is constrained to rotate about a fixed point then the body will undergo and angular acceleration given by $\vec{\tau} = I\vec{\alpha}$ where *I* is the moment of inertia about the axis through the fixed point.

For rotation about the axis of cylindrical symmetry the moment of inertia of a disk is $I_{disk} = \frac{1}{2}MR^2$ and the moment of inertia of a hollow cylinder of finite thickness is $I_{hollow\ cylinder} = \frac{1}{2}M\left(R_1^2 + R_2^2\right)$ where R_1 and R_2 are the inside and outside radii of the hollow cylinder.

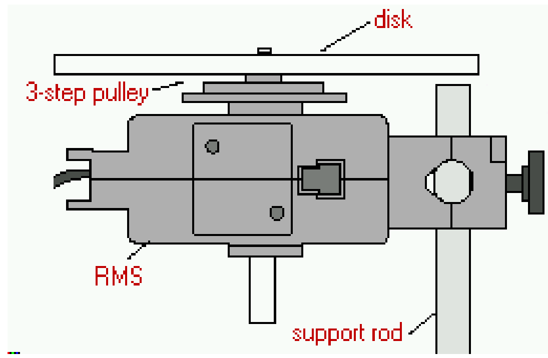
In what follows you should refer to the diagram on the bottom of the next page. Down is positive.

For the mass hanging over the pulley: $mg - T = ma$ where $a = \alpha r$. (Don't be confused by the extra minus sign on both sides of the equation located under the disk in the diagram.)

Substituting for *a* in the first equation gives the tension $T = mg - m\alpha r = m(g - \alpha r)$.

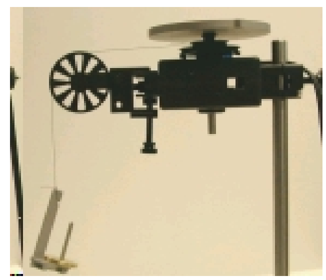
The torque on the disk is: $\tau = rT$ and substituting in the value of the tension from the line above gives $\tau = rm(g - \alpha r)$.

THE EXPERIMENT



Mount the aluminum disk onto the rotary motion sensor above the three step pulley with the square hole in the disk fitting over the square post of the three step pulley. Attach the rotary motion sensor to the support rod of the stand so that it is quite solid..

Place a bubble level on the aluminum disk and level the stand so that the bubble stays in the centre as you rotate the disk. This means that the axis of rotation is vertical.

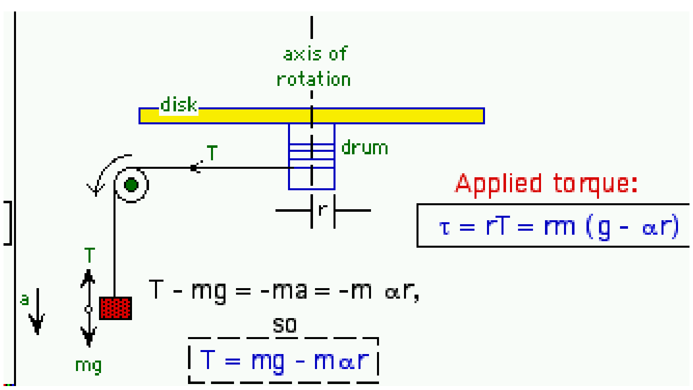


Attach the vertical pulley to the rotary motion sensor with the plastic thumbscrew facing down. **Do not over tighten since the parts are made of plastic and are quite fragile.**

Masses are to be hung from a thread attached to the horizontal three step pulley and this will provide the accelerating torque.

Temporarily remove the aluminum disk, select the middle step of the three step pulley and measure the radius of the drum. At this time you should measure the mass and radius of the aluminum disk.

Attach a thread to the drum of the horizontal pulley by passing the thread through the hole in the pulley and tying a knot. Pass the thread over the vertical pulley and adjust the lateral position of the pulley for the particular drum radius that you have chosen.



Adjust the height of the vertical pulley using the thumbscrew at the side so that the thread passing over the top of the pulley is horizontal as in the diagram. Make sure that the thread is long enough so that the masses can reach the floor while the sensor is still accelerating but not too much longer. How will making the string too long affect your experiment?

Reattach the aluminum disk.

In DataStudio, create an experiment with a rotary motion sensor.

Click on *Setup*.

Double click on the rotary motion sensor and select a *Sample Rate* of 20 Hz from the *General* category, *Angular Velocity* (rad/s) from the *Measurement* category and 360 Division/Rotation from the *Rotary Motion Sensor* category to start. You may investigate other settings later.

Take some preliminary data to observe how the apparatus responds as the masses fall. After the masses reach the floor, stop the rotation to prevent the thread from becoming entangled in the pulley. **Treat the apparatus with care.**

Angular Velocity may be used to set a condition for a delayed start.

MOMENT OF INERTIA OF A DISK

Using the small (approximately 8 gram) masses, measure the angular acceleration for five different masses. Calculate the torque for each run and plot torque versus angular acceleration. The slope of the graph will be the moment of inertia of the system.

Calculate the moment of inertia of the aluminum disk and compare it to the experimentally determined moment of inertia of the system. Is the calculated moment of inertia larger, smaller, or the same as the experimental value? Is this what you would expect? Why or why not?

MOMENT OF INERTIA OF A HOLLOW CYLINDER (OR RING)

Mount the hollow cylinder on top of the disk with the protruding posts sticking into the disk to keep it in place. As above, measure the angular acceleration for five different masses, (use the larger set of masses). Calculate the torque for each run and plot torque versus angular acceleration. The slope of the graph will be the moment of inertia of the system which is the hollow cylinder plus the system for which the moment of inertia was previously determined. By subtracting, determine the moment of inertial of the hollow cylinder.

Measure the mass and dimensions of the hollow cylinder and calculate its moment of inertia. Compare your results to those obtained experimentally.

It should be noted that when you are plotting torque versus angular acceleration you are not plotting two independent variables because the angular acceleration was used in the calculation of the torque. However, they are almost independent since in calculating the torque, αr is small compared to g.

Revised November 6, 2001