Summary of the *Torsion Pendulum* Experiment

This document is a companion to a web-based document on the Torsion Pendulum experiment in the Physics laboratories of the University of Toronto. The web-document may be accessed at:

http://www.upscale.utoronto.ca/IYearLab/Intros/TorsionPend/TorsionPend.html

Since this document is a summary, we do not duplicate the more complete discussion contained in that web-document.

The Relation Between the Period and the Moment of Inertia

$$T^2 = \left[\frac{4\pi^2}{\kappa}\right] I$$

where:

> T is the period.
> I is the moment of inertia
> κ is the torsion constant of the wire

Some Moments of Inertia

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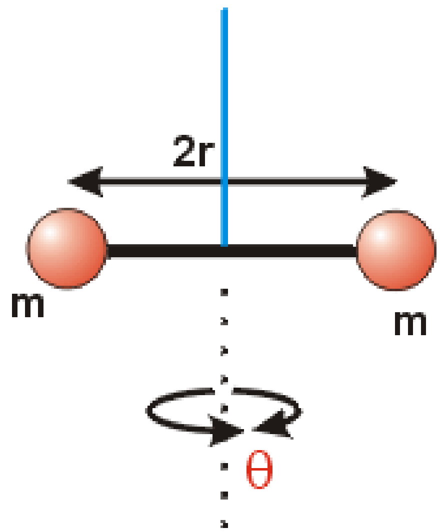
Example 1: Two points masses *m* each a distance *r* from the axis of oscillation, connected by a massless rod.



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$$I = 2\left[m\, r^2\right]$$

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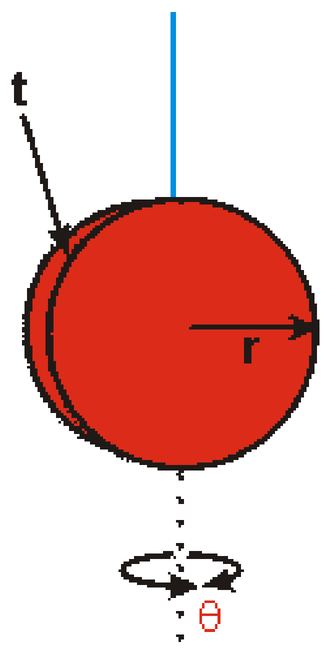
Example 2: A disc of mass *M*, radius *r* and thickness *t* oscillating around the diameter that goes through the center of mass.



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$$I = M\left(\frac{r^2}{4} + \frac{t^2}{12}\right)$$

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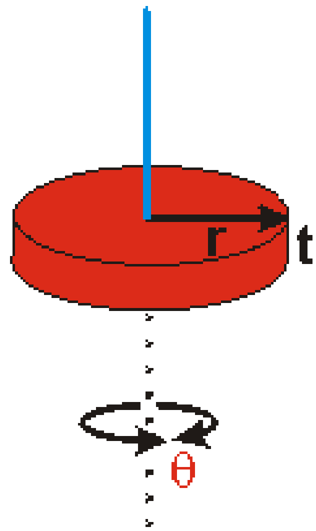
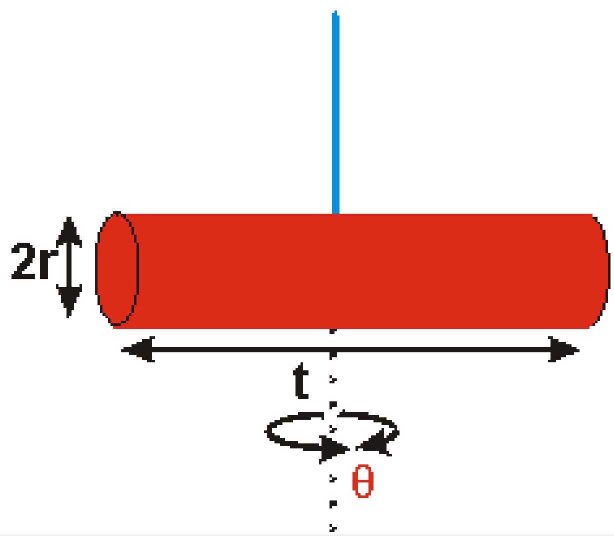
Example 3: The same disc as before, but oscillating around the perpendicular to its face through the center.  	$$I = \frac{Mr^2}{2}$$
Example 4: A cylinder of mass *M*, radius *r* and length *t***.**  	$$I = M\left(\frac{r^2}{4} + \frac{t^2}{12}\right)$$ Note: this is the same result as Example 2 above.

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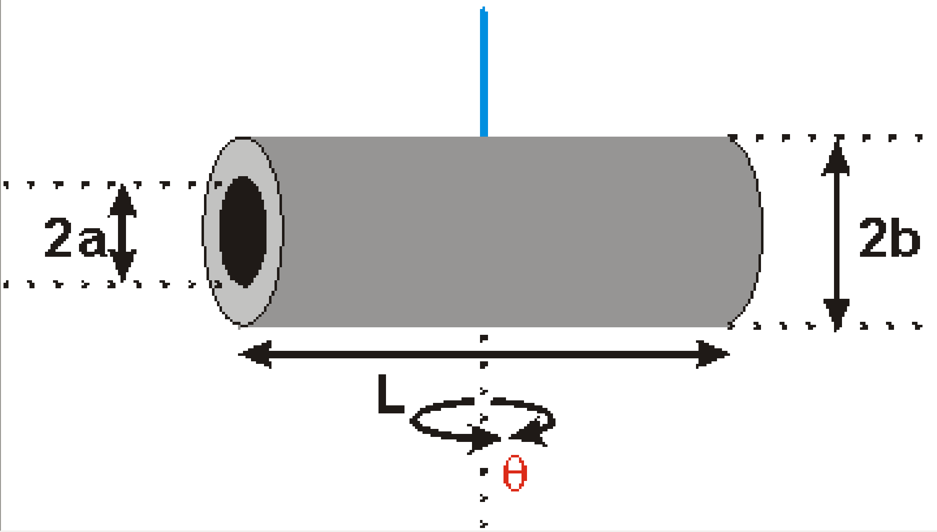
Example 5: A hollow cylinder of mass *M*, length *L*, inner radius *a* and outer radius *b*.



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$$I = M\left[\frac{L^2}{12} + \frac{a^2 + b^2}{4}\right]$$

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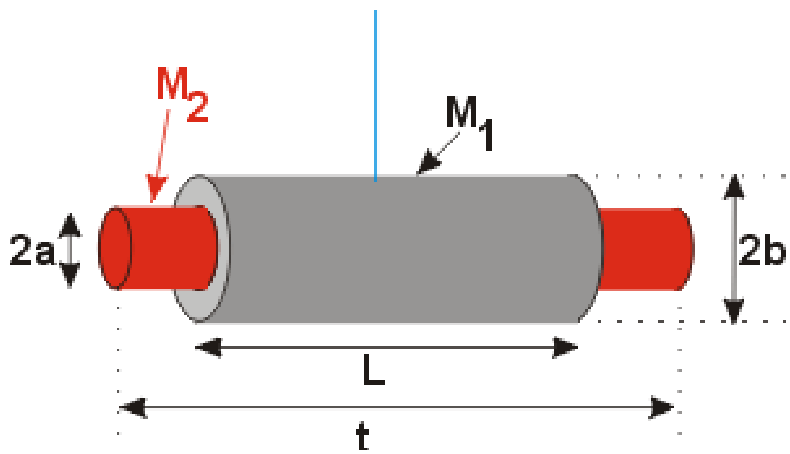
Examples 4 and 5 combined:

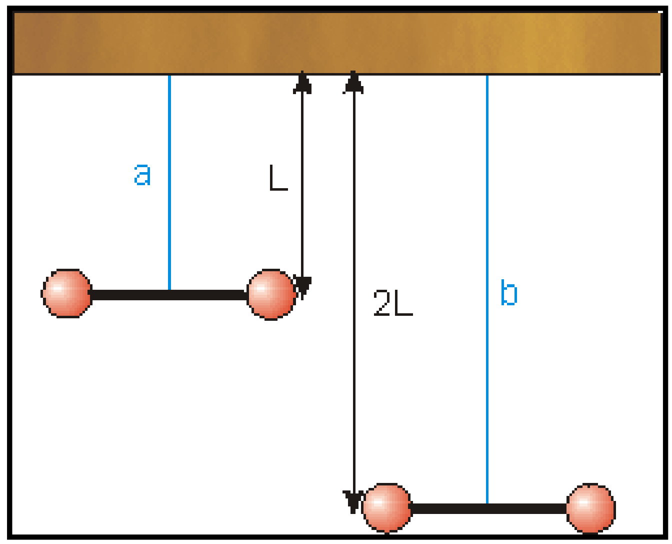


$$I = M_1\left[\frac{L^2}{12} + \frac{a^2 + b^2}{4}\right] + M_2\left(\frac{a^2}{4} + \frac{t^2}{12}\right)$$

Preparatory Questions

- You are doing your laundry. In order to answer the following questions you will need to make some rough estimates of the weight of your wet laundry and the size of the laundry tub in the washing machine.
  - Estimate the moment of inertia of your clothes when the washing machine is in its *spin cycle*.
  - Estimate the uncertainty in your determination of the the moment of inertia of your clothes.
  - Does the moment of inertia change of your clothes change from the beginning to the end of the cycle? Why? By how much?
- Two torsion pendulums are constructed of identical wires except that one wire is twice as long as the other, as shown. The objects hanging from the wires have identical moments of inertia. What is the ratio of the period of oscillation of the two objects:

  T_a / T_b



- In the **Introduction**, some examples of moments of inertia I were given. Consider the moment of inertia of the hollow cylinder of Example 5 as being that of a solid cylinder of radius b minus that of a solid cylinder of radius a, with both cylinders having the same length t. Use the result of Example 4 to derive the result of Example 5. You will wish to use the fact that the mass M of a solid cylinder is:

  $$M = \rho \pi r^2 t$$

  where ρ is its density, r its radius and t its length
- In the experiment you will be measuring the period of oscillation T of a number of objects. One procedure would be to measure the time for 20 oscillations, t_{20}, and repeat the measurement 5 times. Another procedure would be to measure the time for 5 oscillations, t_5, and repeat the measurement 20 times. Assume, reasonably, that the error in the determination of the time for 20 oscillations is the same as the error in the determination of the time for 5 oscillations. Calculate the error in the period for both procedures to determine which will give the smallest error in the value of the period.
  If you have studied the modules on Error Analysis at
  http://www.upscale.utoronto.ca/PVB/Harrison/ErrorAnalysis/ this question may be familiar.

- You are measuring the moment of inertia of a hollow cylinder, as in **Example 5** above. Here are the values:

L	11.72 ± 0.02 cm	Measured with a meter stick
$2a$	15.4 ± 0.3 mm	Measured with a vernier caliper
$2b$	34.2 ± 0.3 mm	Measured with a vernier caliper
M	1289.38 ± 0.04 g	Measured with a balance

  - Convert all the above values to SI units.
  - What is the percentage error in L?
  - What is the value and error in $p = L^2/12$?
  - What is the percentage error in p? How does it compare to the percentage error in L?
  - What is the value, error, and percentage error in a?
  - What is the value and error in a^2?
  - How does the percentage error in a^2 compare to the percentage error in a?
  - What is the value and error in b^2?
  - What is the value and error in $q = (a^2 + b^2)/4$?
  - For the previous calculation, is one of the errors negligible? If yes, which?
  - What is the percentage error in q? Have you calculated any other percentage errors with this value?
  - What is the value and error in $r = p + q$?
  - For the previous calculation, is one of the errors negligible? If yes, which?
  - What is the value and error in the moment of inertia $I = M\,r$?
  - For the previous calculation, is one of the errors negligible? If yes, which?
  - What is the percentage error in I? Have you calculated any other percentage errors with this value?

  Note that the above numbers are fictitious. However, the principles of error propagation and in particular the identification of negligible errors will be used in your analysis. Negligible errors do not even have to be accounted for in real calculations. Thus, by paying attention to the values before pounding numbers into your calculator, you can save much time and effort.