Chaotic Motion
(One or two weights)

Exercises I through IV form the one-weight experiment. Exercises V through VII, completed after Exercises I-IV, add one weight more.

This challenging experiment demands your independent study and creativity. Don’t rely on the Lab Demonstrator – discover the unknown phenomenon on your own! Be a pioneer!

The recommended textbooks are:
Physics for Scientists and Engineers by R.A.Serway and J.W.Jewett (Jr.), Volume I and Physics for Scientists and Engineer with Modern Physics by R.D.Knight, Volume I.

INTRODUCTION
Chaotic behavior has been observed in the laboratory in electrical circuits, lasers, oscillating chemical reactions, fluid dynamics, and mechanical and magneto-mechanical devices. Observations of chaotic behavior in nature include the dynamics of satellites in the solar system, the time evolution of the magnetic field of celestial bodies, population growth in ecology, the dynamics of the action potentials in neurons, and molecular vibrations. Everyday examples of chaotic systems include weather and climate. A related field of physics called quantum chaos theory studies systems that follow the laws of quantum mechanics. Recently, another field, called relativistic chaos, has emerged to describe systems that follow the laws of general relativity.

The first discoverer of chaos was Henri Poincaré. In 1890, while studying the three-body problem, he found that there could be orbits which were nonperiodic, and yet not forever increasing nor approaching a fixed point.

Despite initial insights in the first half of the twentieth century, chaos theory became formalized as such only after mid-century, when it first became evident for some scientists that linear theory, the prevailing system theory at that time, simply could not explain the observed behavior of certain experiments. What had been beforehand excluded as measure imprecision and simple "noise" was considered by chaos theories as a full component of the studied systems. The main catalyst for the development of chaos theory was the electronic computer.

For a dynamical system to be classified as chaotic, it must be sensitive to initial conditions. As a result of this sensitivity, which manifests itself as an exponential growth of perturbations in the initial conditions, the behavior of chaotic systems appears to be random. This happens even though these systems are deterministic, meaning that their future dynamics are fully defined by their initial conditions, with no random elements involved. This behavior is known as deterministic chaos, or simply chaos.

"Chaos", a journal published by the American Institute of Physics starting 1991 and devoted to increasing the understanding of nonlinear phenomena, writes: “Building on important but isolated historical precedents (such as the work of Poincaré), "chaos" has in some cases caused a fundamental reassessment of the way in which we view the physical world. For instance, certain seemingly simple natural nonlinear processes, for which the laws of motion are known and completely deterministic, can exhibit enormously complex behavior, often appearing as if they were evolving under random forces rather than deterministic laws. One consequence is the remarkable result that these processes,
although completely deterministic, are essentially unpredictable for long times. The same type of "deterministic chaos" can be observed, for example, in electrical activity from biological systems; in the transition of a fluid to turbulent motion; and in the motion of the moons of the giant planets.”

You could observe the chaotic motion of a swinging spring in the mandatory experiment “Simple Harmonic Motion”. The simulation of the chaotic motion of the swinging spring can be found at http://mathsci.ucd.ie/~plynch/SwingingSpring/chaoticspring.html

Another example of the chaotic motion is on your desk: it is a pendulum with magnets at the base. You can study its motion in detail at the end of your experiment applying skills and using concepts obtained during the performance of the Chaotic Motion experiment.

In your experiment, you will investigate the influence of a number of variables on the chaotic motion of the rotating oscillator. The oscillator consists of an aluminum disk connected to two springs and a driver. A point mass on the edge of the aluminum disk makes the oscillator nonlinear. Several quantities can be varied to cause regular motion to become chaotic. These variables are the driving frequency, driving amplitude, damping amplitude, and the initial position of the point mass.

**EQUIPMENT AND SOFTWARE**

The setup is shown on Fig.1.

The oscillator consists of an *aluminum disk* (1) connected to *two springs* (2). The disk is coaxial with the *rotary motion sensor* (3), which must be plugged into channels 1 and 2 on the Science Workshop 750 interface – the PASCO box (4). A *point mass* (5) on the edge of the aluminum disk makes the oscillator nonlinear. This nonlinearity is required to cause chaotic motion. The disk can be magnetically damped turning the screw (6) with a *magnet* behind the disk toward or away from the disk. One of the springs is tied to the arm of the *mechanical oscillator/driver* (7), which is connected to a PASCO box (4). The power supply of the PASCO box provides rotating of the arm of the driver which crosses and interrupts the beam of the photogate (8), plugged into the channel 3 of the PASCO box. The photogate is used as a timer and the counter of the number of oscillations.

The driver is plugged into the PASCO box with the black socket grounded (┴). The red wire of the driver must be plugged into the red socket. On the Setup page “Add sensors and instruments” a Signal generator must be added to output channels, and 5.000V must be set up by pressing + button on the window.
The frequency of the sinusoidal driver can be varied to investigate the progression from predictable motion to chaotic motion. Magnetic damping can be adjusted to change the character of the chaotic motion. The angular position and velocity of the disk are recorded as a function of time using a Rotary Motion Sensor.

You will use the DataStudio file CHAOS.ds. The file can be opened as follows:
My Computer » Local Disk (C:) » Program Files » PASCO Scientific » DataStudio » Chaos » DataStudio Files » CHAOS

Open the file CHAOS.ds and study all settings, functions and fields prepared for plotting graphs.

There are three different ways of plotting oscillations:

1. Angular Displacement ($\theta$) vs. time
2. Phase Space: Angular Velocity ($\omega$) vs. Angular Displacement ($\theta$)
3. Poincaré Plot: Angular Velocity ($\omega$) vs. Angular Displacement ($\theta$) plotted only once per period of the driving force.

The phase space and the Poincaré plot are particularly useful for recognizing chaotic oscillations. When the motion is chaotic, the graphs do not repeat. The Poincaré plot is graphed in real time, superimposed on the phase plot. This is achieved by recording the point on the phase plot once every cycle of the driver arm as the driver arm blocks a photogate.

**Exercise I: Superimposing the Plots on the Phase Diagram**
This exercise is performed with the file Chaos small.ds. This file contains the graphs obtained in some experiment for Poincaré plots and phase diagram. First, you will make the diagram grey. The steps are:

1) Left double click on the function “Angular Velocity, Ch3&4 vs Angular Position, Ch 3&4 (rad/s)”.
2) In the opened window, choose Appearance option
3) Click on the “Change Color”
4) Select the grey color and click OK twice to exit the window

Now, you will work with the field of the phase diagram:
5) Right click on any place in the diagram
6) Click Settings at the very bottom of the menu
7) Select Layout in the upper row of options
8) Find and select: “Create New Graph” and “Unit of Measure” without changing the other settings. Then, OK.
9) Return to the left column under the title “Data”. Left click and drag the Run # under the function “$v$ vs $x$ (rad/s)” down to the Display of Phase Plot. As a result, you will see the plots in the phase diagram connected by lines which is not needed.
10) Right click on the field of the diagram, then click Settings at the bottom of the menu
11) At the top of the appeared window, delete “Apply to all”, and select “$v$ vs $x$, Run #”
12) In the option Appearance, delete “Connect data Points” and select “Show Data Points”.
13) Click OK
14) Enjoy your diagram

PART I.

Exercise II: Potential Well

Before starting the exercise, check whether the setup is ready for your experiment. The string must have a length of 1.5 m. The center of the string must be tied around the smallest step of the Rotary Motion Sensor pulley. Both ends of the string must be threaded through the side hole on the largest step of the pulley. Each end of the string must be wrapped once around the largest step of the pulley. The disk must be able to rotate one revolution in either direction without the end of either spring hitting the pulley. Also neither spring should completely close. The point mass and the centre of the disk should not be aligned along one vertical.

The exercise permits you to study the function of potential energy and to find an equilibrium point or points if many for the pendulum. To map the potential energy, $U$, versus the angle, $\Theta$, that the pendulum point mass is displaced from vertical, the magnetic damping and the driving force are removed and the pendulum is displaced from vertical and allowed to oscillate freely. The angular velocity is measured, and thus the kinetic energy ($K$) can be calculated. Then the potential energy $U$ is derived from the law of conservation of energy:

$$U_i + K_i = U + K = c$$

Since the pendulum starts from rest at maximum displacement, $K_i = 0$ when $U = U_i$. The oscillating disk can be considered a rotating rigid body [1: p.277; 2: p.346], so that the function of its potential energy must depend on its angular frequency as follows:

$$U = c - \frac{1}{2} I \omega^2$$

Therefore, the shape of the potential energy well can be found by plotting the negative of the square of the angular speed ($-\omega^2$) versus the angular displacement ($\Theta$).

Procedure:
- Leave the driver power supply turned off. Screw the magnet screw all the way back away from the disk to reduce the drag. Displace the point mass to one side far enough that the disk will oscillate all the way over to the other side when it is released.
- Click on START in DataStudio, release the pendulum, and let it oscillate once. Then click on STOP.
- Examine the resulting plot of potential energy versus angle. Print it out.
Questions:
1. How many equilibrium points do you see on the graph? What points refer to stable equilibrium? Verify your answer experimentally and explain the method of verification.
2. If you have found more than one potential well, are the wells equally deep? Why or why not?

Exercise III: Resonant Frequency

Upon the definition [1: p.453; 2: pp.431-432], the resonance is the dramatic increase in amplitude of oscillations of the forced, or driven, oscillator at the frequency of the driving force near the natural frequency \( \omega_0 \) of the undamped oscillator. The natural frequency \( \omega_0 \) is also called the resonant frequency. The true natural frequency cannot be measured directly in the real experiment by measuring the frequency of damped oscillations [1: p.452; 2: p.428].

Procedure and Questions
- Screw the magnet toward the disk until it is about 3 mm from the disk. Without turning on the power supply that powers the driver, allow the point mass to fall into the equilibrium position on either side of the pendulum. Click on START, displace the pendulum from equilibrium and let it oscillate for a few oscillations. Click on STOP.
- Examine the angle vs. time graph. Are the oscillations sinusoidal? Are they damped?
- Examine the phase plot (angular speed vs. angle). What shape is it? How is it affected by the amount of damping? What would it look like if there were not any damping? (To answer the question, try to decrease damping carefully turning the screw away from the disk. Don’t eliminate damping completely!)
- Measure the period of the oscillation using the Smart Tool at the top of the angle vs. time graph.
- Using equation (15.32) [1] or equation (14.56) [2], the angle vs. time graph obtained in your experiment and the formula for frequency \( \omega \) of the damped oscillation
  \[
  \omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2},
  \]
  calculate the natural frequency \( \omega_0 \) of the oscillator.

Exercise IV: Non-Chaotic Oscillations

NOTE ABOUT INITIAL CONDITIONS: For the rest of the experiment, hold the point mass end at the top and then let go when the driver arm is at its lowest point. Don’t permit forced oscillations without damping. This can tangle threads.

Procedure and Questions
- Set the driver arm for amplitude of about 3 – 4 cm. Make sure the driver arm only breaks the photogate beam once per revolution. Adjust the magnet distance to about 4 mm from the disk. Turn on the power supply and adjust the voltage to about 3 V so the oscillation is simply one
back-and-forth motion. You should make the adjustment very carefully.

- Click on START and record data for a few minutes.

- Examine the graph of angle vs. time. Is it sinusoidal? What is the period? Is the period the same as the driving period (take into account the error)? Why is this graph different from the graph in Part III?

- Examine the graph of angular velocity vs. angle (the phase diagram). Explain the shape of the graph. How is it different from the phase diagram in Part III?

- Examine the Poincaré plot. This graph must have no connecting lines. If they appear, open SETTINGS (the last to the right hand side of the row) on the graph and choose NO LINE option. Explain the shape of the graph. How does this plot indicate that this oscillation is regular?

Superimposing Poincaré Plot on Phase Diagram:
To superimpose the Poincaré plot on the phase diagram, open the Graph Settings window on the phase graph. Verify under "Appearance" that Full Color is not selected. On "Layout" click on Create New Graph and Group by Unit of Measure. Click and drag the v vs. x data onto the phase graph. Then select the v vs. x data on the graph and choose to plot data points but no connecting line. The phase data should appear in gray and should have connecting lines on. The v vs. x data should be in color and superimposed on the phase data. To return to the original configuration, you can reverse the steps taken above, or simply re-open the original file after saving your data.

- Gradually and carefully increase the driving frequency by increasing the voltage on “power supply” to +6V. Give the pendulum time to respond to the change in driving frequency. Increase the frequency until the motion of the pendulum is slightly more complicated: It should not simply have one back-and-forth movement but rather it should oscillate back-and-forth with an extra back-and-forth movement on one side. Re-start the oscillation, holding the point mass end at the top and letting go when the driver arm is at its lowest point.

- Click on START and record data for a few minutes.

- Examine the graph of angle vs. time. Is it sinusoidal? What is the period? Is the period the same as the driving period? How is it different from the previous oscillation?

- Examine the graph of angular velocity vs. angle (the phase diagram). Explain the shape of the graph. Compare it to the previous phase diagram.

- Examine the Poincaré plot. Explain the shape of the graph. How does this plot indicate that this oscillation is regular?
PART II.

To perform and interpret the experiments of the PART II you will need the results obtained in the previous exercises. Be sure that the setup you are using for the second part of the experiment is same. Try to measure briefly the frequency of the oscillator to verify the properties of the oscillator. The driver will be used in all exercises of this part of the experiment.

The driving frequency, the driving amplitude and the position of the magnet will be used as variables to transform the regular oscillations into chaotic motion.

Exercise V: Chaotic Oscillations. Varying the Driving Frequency
Check settings for the power supply, the length of the driver arm and position of the magnet to bring them to same values and positions that you used at the end of the Exercise IV performance.

Procedure and Questions
- Start with applied voltage of about 3V and gradually increase the driving frequency to the resonant frequency by increasing the voltage on the power supply. To make the motion of the pendulum very complicated, you may have to adjust the distance of the magnet from the disk. The pendulum should pause suddenly at various points in its motion and spend random times on each side of the oscillation. Re-start the oscillation, holding the point mass end at the top and letting go when the driver arm is at its lowest point.

- Click on START and record data for an hour.

- Examine the graph of angle vs. time. Is it sinusoidal? What is the period? Is the period the same as the driving period (compare with errors)?

- Examine the graph of angular velocity vs. angle (the phase diagram). Explain the shape of the graph.

- Examine the Poincaré plot. Explain the shape of the graph. How does this plot indicate that this oscillation is chaotic?

Exercise VI: Chaotic Oscillations. Varying Magnetic Damping and the Driving Amplitude
The driving frequency was varied to change the oscillation from regular to chaotic. Try adjusting the magnetic damping while holding the driving frequency at the frequency that gave chaos before.

Then try holding the damping and driving frequency constant while varying the driving amplitude. The driving amplitude may be changed by adjusting the driver arm at different positions. The minimum amplitude was set up in the Exercise IV.

Questions:
1. What special variable did you choose to demonstrate the transformation of oscillation into chaotic motion as a process dependent on this variable while adjusting the magnetic damping only? How does this variable change the phase diagram pattern and the Poincaré plot?
2. Does amplitude, set-up with the driving arm, influence the oscillator behavior in same principal way for different frequencies and positions of the magnet? Why or why not?
Exercise VII: Chaotic Oscillations. Varying the Initial Position of the Point Mass

This is the most challenging experiment as you have to set up the driving frequency, the driving amplitude, the position of the magnet in the way that can provide evidence of what was stated in the Report [3]: "Chaos in mathematics means extreme sensitivity to initial conditions, i.e., minute differences in the initial conditions are amplified exponentially. Given an initial condition, the dynamic equation determines the dynamic process, i.e., every step in the evolution. However, the initial condition, when magnified, reveals a cluster of values within a certain error bound. For a **regular** dynamic system, processes issuing from the cluster are bundled together, and the bundle constitutes a predictable process with an error bound similar to that of the initial condition. In a **chaotic** dynamic system, processes issuing from the cluster diverge from each other exponentially, and after a while the error becomes so large that the dynamic equation loses its predictive power."

Check the effect of initial position of the point mass on the oscillations and appearance of chaos.

- What variable have you chosen to characterize the initial position of the point mass?
- Expose 4-6 best graphs to approve or disapprove the statement about the sensitivity of the chaotically oscillating system upon the initial position of the point mass.
- Can the Brownian motion of particles of suspended matter, discovered with an optical microscope and explained by A. Einstein’s theory in 1905 [1: p.543], be called the *chaotic motion*?

References

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by Natalia Krasnopolskaia