Uncertainty in Physical Measurements: Module 0 – Introduction

To the Student

A crucial part of the way science describes the physical universe is quantitative. In general there are two different types of numbers that we use in the description of some physical system:

1. An exact value. For example, there are exactly three bananas shown to the right.
2. An approximate value. If we want to determine the total mass of the bananas we will need to use some sort of scale. However, in the real world there are no perfect scales, so the measured mass is always an approximate value.

Say we measure the mass of the bunch of bananas with a digital scale, and the reading on the scale is 324.1 grams. The question is what is the **uncertainty** associated with the value of 324.1?

The answer to this type of question often depends on the context. For example, if a carpenter says that some length is "just 8 inches," she probably means that she thinks the length is closer to $8\frac{0}{16}$ inches than it is to $7\frac{15}{16}$ inches or $8\frac{1}{16}$ inches. If a machinist says a length is "just 200 millimeters" he probably means that he thinks the length is closer to 200.00 mm than it is to 199.95 mm or 200.05 mm.

In general, the best way to describe the uncertainty in some measurement is to assign it a quantitative value. The following Modules concentrate on how to determine and communicate such a number. Two examples may help make it clear to you why this material is important to anybody doing in work in the sciences, whether or not the science is Physics.

Example 1

Sadly, many otherwise good scientists are unaware of the material you will be learning here, which sometimes causes some wrong conclusions.

For example, in the early 1970's some researchers reported that a diet that was high in fiber reduced the incidence of *polyps* in the colon. Polyps are a precursor to cancer, so people began cramming as much fiber down their gullet as they could.

In January 2000 a massive study in the New England Journal of Medicine showed that fiber in the diet has no effect on the incidence of polyps.

There were two problems with the earlier study: First they had a fairly small number of people in the two samples they studied. Second, they neglected to do an analysis of the uncertainties in the results based on the sample sizes.

If they had calculated the uncertainties in their numbers, they would have known that although the measured rates of polyp formation were different for the two groups, those differences were zero within the uncertainties of the measured rates.

Note that there are benefits to fiber in your diet, but not as a way to reduce the formation of polyps in your colon.

Example 2

As you may know, the *neutrino* is one of the elementary particles that make up the universe. Between 2009 and 2011, in a very complex and difficult experiment, a collaboration of 178 physicists based in CERN in Switzerland and the Laboratori Nazionali del Gran Sasso in Italy measured the speed of 16,111 neutrinos, using an instrument called the *Oscillation Project with Emulsion-tRacking Apparatus* (OPERA). The neutrinos travelled from CERN to the lab in Italy, a distance of 721278.0 m. Although the measured speeds of the individual neutrinos varied, the mean value of all speeds was:

$$\overline{v} = (1.000\,024\,8)c > c \tag{1}$$

where c is the speed of light in a vacuum.

According to Einstein's 1905 *Special Theory of Relativity*, no object can move faster than the speed of light. So if this experimental result is correct, then that theory must be wrong! Therefore it is crucial to know what the uncertainty is in the number of Eqn. 1.

Example 2 continued

Because the discrepancy from the speed of light occurs in the one-hundred-thousandths position, it is easier for us humans to "read" the experimental result by defining t_c as the time for an object to travel the distance at exactly the speed of light and $\overline{t}_m$ to be the mean measured time of the neutrinos. Then we define $\delta t = t_c - \overline{t}_m$ and:

$$\delta t \begin{cases} < 0, & v < c \\ = 0, & v = c \\ > 0, & v > c \end{cases} \tag{2}$$

Using this notation, the experimental result is:

$$\delta t = 60 \text{ ns} \tag{3}$$

If the uncertainty in the value of δt is, say, 500 ns, then the experimental result is consistent with the theory of relativity: the data only tells us is that the value of δt is probably between (60 – 500) = −440 ns and (60 + 500) = +560 ns. So the actual value of δt could well be a negative number and it could well be that neutrinos do not travel faster than *c*.

However, when the experimenters evaluated all the individual uncertainties in all the various parts of the experiment they calculated that the total uncertainty was 10 ns, so the actual value of δt is probably between (60 – 10) = 50 ns and (60 + 10) = 70 ns. We write this result as:

$$\delta t_{\text{OPERA}} = (60 \pm 10) \text{ ns} \tag{4}$$

This shocking result caused the experimenters to spend months re-checking everything they could think of in the experiment. They found no mistakes so in September 2011 they published their result, which made headlines around the world. Being excellent scientists, they also released all of their raw data and invited others to check their work.

Simultaneously to the OPERA experiment, another team at CERN and Italy was doing measurements of the speed of neutrinos. The experiment was called *Imaging Cosmic And Rare Underground Signals* (ICARUS). In March 2012 they published their final result, which was:

$$\delta t_{ICARUS} = (0.3 \pm 9.8) \text{ ns} \tag{5}$$

This result is completely consistent with neutrinos travelling at less than or equal to the speed of light.

Example 2 continued

Later, the OPERA team discovered a loose fiber optic cable and a wrongly functioning oscillator. When they revised their calculations, they published a new result:

$$\delta t_{\text{OPERA, revised}} = (6 \pm 11)\ \text{ns} \tag{6}$$

This new result is consistent with the ICARUS one within experimental uncertainties. So, at least for now, the Special Theory of Relativity has been saved.

Note that the experiments have not proved that the Special Theory is correct. They have also not proved that neutrinos do not travel faster than the speed of light. All they have shown is that the data are consistent with neutrinos travelling at speeds < *c*.

You may have noticed above that we were careful to use the word *probably* in, for example, "… the actual value of δt is probably between (60 – 10) = 50 ns and (60 + 10) = 70 ns." This indicates, correctly, that our study of uncertainty in physical measurements will require understanding some elementary statistics. We will begin that study in Module 1.

Each Module contains the material that you need to know, and some Questions and Activities. You should definitely read the materials before you come to the Practical. In the Practical you will be working with your classmates in a small Team to answer the Questions and do the Activities. Reading the Questions before is a good idea, so that you and your classmates will already have an idea of what they are about.

Enjoy!

To the Instructor

These Modules are based on the *Guide to the Expression of Uncertainty in Measurement*, commonly called just *GUM*. Development of *GUM* began in1993. It has been adopted by:

- All major national measurement institutes, including the Canadian *Institute for National Measurement Standards*.
- ISO 17025, which is required for accreditation by the *International Laboratory Accreditation Standards* organization.
- Most government and industry research laboratories around the world.

Surprisingly, it is still little known in Universities.

GUM provides a probabilistic framework which is consistent and logical for all experimental uncertainties. This makes it preferable to traditional approaches to teaching this subject, which is rife with logical inconsistencies that only serve to further confuse our students.

Another issue with the traditional approach to teaching this subject involves nomenclature. The traditional approach discusses the assignment of "errors" to physical measurements. The word error implies that some mistake has been made. But most uncertainties in physical measurements do not involve any mistakes, but arise because of the limitations in our measuring instruments. Thus, in these Modules the word "error" does not appear except in the context of systematic effects. We urge you to similarly banish the word from your conversations with your students.

Modules 0 – 4 are the "core" of the material. Module 5 is mostly concerned with fitting of data to models, and Module 6 discusses miscellaneous topics that didn't fit nicely into the previous ones.

The Modules were designed for undergraduate students in the sciences. They contain the information the students need to learn, and a number of Questions and Activities. Most of the Activities involve the use of simple apparatus. The Questions and Activities are intended to be worked on by a small Team of 2 – 4 students in the Lab or Practical.

It is important for the students to have read the Module before coming to the Lab / Practical where they will work on the Questions and Activities. We have prepared a bank of questions for Reading Quizzes, which can be given beforehand. Especially if they count for a small mark, they will help motivate the students to actually do the readings. The questions are designed to be almost trivially easy if the student has actually read the material. Typically 3 or 4 questions are asked. The questions in the bank are available to Instructors by contacting the author.

Each Module also has an accompanying Instructor Guide, which should not be available to the students. Instructors may access these Guides by contacting the author.

If you wish to learn more about this approach to teaching uncertainties, here are some references to help get you started:

- S. Allie, A. Buffler, B. Campbell, F. Lubben, D. Evangelinos, D. Psillos, and O. Valassiades, "Teaching Measurement in the Introductory Physics Laboratory," The Physics Teacher **41** (2003), 394.

- Buffler, S. Allie, and F. Lubben, "Teaching Measurement and Uncertainty the GUM Way," The Physics Teacher **46** (2008), 539.

- S. Pillay, A. Buffler, F. Lubben, and S. Allie, "Effectiveness of a GUM-compliant course for teaching measurement in the introductory physics laboratory," European Journal of Physics **29** (2008), 647.

- A. Possolo and B. Toman, "Tutorial for Metrologists on the probabilistic and statistical apparatus underlying the GUM and related documents," National Institute of Standards and Technology (NIST), 2011: http://www.nist.gov/itl/sed/gsg/upload/TutorialMetrologists2011Nov22.pdf

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