Solutions: Term Test #1

1) i) Displacement is a vector and refers to the difference in position vectors at two different times. Distance refers to a scalar quantity related to the total path traveled in the interval for which one adds the magnitudes of the infinitesmal displacements related to the time interval.

ii) Since Newton's laws are valid only in frames of reference which are inertial frames moving with constant relative velocity the anwer is no. In an inertial frame all accelerations are related to physical forces. In another frame moving with constant acceleration, one would have to correct Newton's laws with "fictitious" forces related to the relative acceleration.

iii) It's acceleration can be zero if its instantaneous angular acceleration and velocity are both zero since the former will make the tangential component of acceleration equal to zero, the latter will make the radial component equal to zero. This can occur, *e.g.,* if the object is momentarily at rest and also not experiencing angular acceleration. If you interpreted "moves" to mean constantly moving with uniform velocity (an interpretation I'll allow) then at least the radial (centripetal acceleration) would be nonzero.

iv) The period of a mass on a spring depends only on the ratio of spring constant and mass and does not involve gravity.

v) The displacement of an object executing simple harmonic motion varies as the cosine (or sine) of an argument. That argument is referred to as phase angle and dictates where the object is and how it is moving within a cycle.

2. Take the positive x-direction to be down the plane. Take the positive y direction to be perpendicular to the plane, on the upside. The normal force on the aunt box is $m_1 g\cos 30^0$ (along y) and the frictional force (along x) is $-\mu_1 m_1 g\cos 30^0$. The total component of the force along the plane (including the component of its weight and the tension in the rod) on the aunt box is $F_x^{aunts} = T + m_1 g\sin 30^0 - \mu_1 m_1 g\cos 30^0$. Similarly for the uncles $F_x^{uncles} = -T + m_2 g\sin 30^0 - \mu_2 m_2 g\cos 30^0$. The total force on the system as a whole is

$$F_x^{system} = F_x^{aunts} + F_x^{uncles} = m_1 g\sin 30^0 + m_2 g\sin 30^0 - \mu_1 m_1 g\cos 30^0 - \mu_2 m_2 g\cos 30^0 = (m_1 + m_2)a_x^{system}$$

from which

$$\frac{m_1 \sin 30^0 + m_2 \sin 30^0 - \mu_1 m_1 \cos 30^0 - \mu_2 m_2 \cos 30^0}{m_1 + m_2} g = a_x^{system} = a_x^{aunts} = a_x^{uncles}.$$ This gives $a_x^{system} = a_x^{aunts} = a_x^{uncles} = 0.37g = 3.7 m/s^2$. And from $F_x^{aunts} = T + m_1 g\sin 30^0 - \mu_1 m_1 g\cos 30^0 = m_1 a_x^{aunts}$ we get $T = 6.1 - 8.25 + 3.3 = 1.2\ N$.

If the uncles and aunts are interchanged, the uncles will push on the rod with force, T so the rod will now be pushing back on the uncles with force *-T*, along the plane; similarly the rod will be pushing on the aunts with force, T; However, the equations above do not change so the acceleration of the boxes down the plane is the same, as is the net force on each box.

3. i) We have $\vec{v} = \frac{d\vec{r}}{dt} = R\frac{d\hat{r}}{dt} = R\frac{d\theta}{dt}\frac{d\hat{r}}{d\theta} = R(\frac{2\pi}{T})\hat{\theta}$.

ii) Similarly $\vec{a} = \frac{d\vec{v}}{dt} = R(\frac{2\pi}{T})\frac{d\hat{\theta}}{dt} = R(\frac{2\pi}{T})\frac{d\theta}{dt}\frac{d\hat{\theta}}{d\theta} = -R(\frac{2\pi}{T})^2\hat{r}$

iii) If the radius is not constant we have: $\vec{v} = \frac{d\vec{r}}{dt} = r\frac{d\hat{r}}{dt} + \frac{dr}{dt}\hat{r} = r\hat{\theta}\frac{d\theta}{dt} + \frac{dr}{dt}\hat{r}$ and

$\vec{a} = \frac{d\vec{v}}{dt} = r\hat{\theta}\frac{d^2\theta}{dt^2} + r\frac{d\hat{\theta}}{dt}\frac{d\theta}{dt} + \frac{dr}{dt}\hat{\theta}\frac{d\theta}{dt} + \frac{dr}{dt}\frac{d\hat{r}}{dt} + \frac{d^2r}{dt^2}\hat{r} =$

$(r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt})\hat{\theta} + (-r\left(\frac{d\theta}{dt}\right)^2 + \frac{d^2r}{dt^2})\hat{r}$. Hence the radial component of the acceleration is $(-r\left(\frac{d\theta}{dt}\right)^2 + \frac{d^2r}{dt^2})\hat{r}$.

4) Take the horizontal direction to be along x and the vertical direction to be along y. Take the launch time to be zero time. We have that $v_x = v_0\cos 60^0$ and $v_y = v_0\sin 60^0 - gt$ where v_0 =100 *m/s*. Hence the object reaches maximum height when $v_y = 0$ or $t = v_0\sin 60^0/g = 5\sqrt{3}$ *s*. The horizontal distance traveled is $v_x t = 250\sqrt{3}$ m and the maximum height is $v_y t - \frac{1}{2}gt^2 = (50\sqrt{3})(5\sqrt{3}) - \frac{1}{2}(10)(5\sqrt{3})^2$=375 *m*. The horizontal angle defined by the velocity vector is $\tan 45^0 = 1 = \frac{v_y}{v_x} = \frac{50\sqrt{3} - 10t}{50}$. Hence t = $5\sqrt{3} - 5$=*3.7s.*

The velocity of the object just before explodes is 50 *m/s* (along x). When the object explodes, by Galilean relativity each object moves with horizontal and vertical components of velocity equal to $100\cos\alpha + 50$ and $100\sin\alpha$. At any time later, these are $100\cos\alpha + 50$ and $100\sin\alpha - gt$. When a fragment hits the ground $y = 0 = 375 + (100\sin\alpha)t - \frac{1}{2}gt^2$ or $t = \frac{100\sin\alpha + \sqrt{(100\sin\alpha)^2 + 750g}}{g}$ (taking positive root). At this time the angle made by horizontal and vertical components of velocity is given by $\tan\theta = \frac{100\sin\alpha - gt}{100\cos\alpha + 50} = \frac{-\sqrt{(100\sin\alpha)^2 + 750g}}{100\cos\alpha + 50}$. If $\alpha = \pi/2$, θ =–70⁰ (–1.2 rad).