I. PROBLEM-1

A.

The equation of the hypotenuse is $y = -\frac{x}{2} + 15$ when the coordinate system is chosen such that the origin is at the bottom of the sign on the axis. Using this the moment of inertia is given by $I = \int r^2 dm = \int_0^{30} \frac{M}{A} x^2 y dx$ where A is the area of the triangle. The moment of inertia is given by $I = 3 * 10^{-2} kgm^2$.

B.

Use $FR = I\alpha$ for $R = 0.05m$ and it is found that $\alpha = 5rads^{-2}$. The angular displacement at $2s$ is $\theta_2 = 1/2\alpha t^2 = 10rad$. The angular velocity at $t = 2s$ is $\omega_2 = 10rads^{-1}$. Since ω is constant after $t = 2s$ the angular displacement between $2s$ and $4s$ is $\theta_4 - \theta_2 = 20rad$. The total displacement is $30rad$. Hence the number of revolutions completed is ≈ 5.

C.

Kinetic energy at $t = 2s$ is $1/2I\omega_2^2 = 1.5J$. The power at $t = 1s$ is $P = \tau\omega_1 = FR\omega_1 = 0.75W$

II. PROBLEM-2

The effective spring constant $k = (4+4)N/m = 8N/m$ since the springs are parallel. Let x be displacement at any time t from the initial position along the plane. Using conservation of energy we have

$$mg\sin(\theta)x = \frac{1}{2}kx^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}m(\frac{dx}{dt})^2 \tag{1}$$

Use $\omega = 1/R\frac{dx}{dt}$ and differentiate Eq(1) with respect to time to obtain the dynamical equation. We find

$$mg\sin(\theta)\frac{dx}{dt} = kx\frac{dx}{dt} + (\frac{I}{R^2} + m)\frac{dx}{dt}\frac{d^2x}{dt^2} \tag{2}$$

So the equation of motion is

$$\frac{d^2x}{dt^2} = \frac{mg\sin(\theta)}{\frac{I}{R^2} + m} - \frac{k}{\frac{I}{R^2} + m}x \tag{3}$$

A change of variables $x' = x - \frac{mg\sin(\theta)}{k}$ transforms the equation into that of a simple harmonic oscillator. Hence the solution of Eq(3) is

$$x(t) = \frac{mg\sin(\theta)}{k} + A\cos(\sqrt{\frac{k}{\frac{I}{R^2}+m}}t + \Phi) \tag{4}$$

where A and Φ are determined by the initial conditions. Using the fact that at $t = 0$, $x = 0$ and $dx/dt = 0$, the final solution can be written as

$$x(t) = \frac{mg\sin(\theta)}{k}(1 - \cos(\sqrt{\frac{k}{\frac{I}{R^2}+m}}t)) \tag{5}$$

The velocity can be obtained by differentiating Eq(5) and is given by

$$\frac{dx(t)}{dt} = \frac{mg\sin(\theta)}{k}\sqrt{\frac{k}{\frac{I}{R^2}+m}}\sin(\sqrt{\frac{k}{\frac{I}{R^2}+m}}t) \tag{6}$$

A.

$\frac{dx}{dt}$ is maximum at $t_{max} = \frac{\pi}{2\sqrt{\frac{k}{\frac{I}{R^2}+m}}}$. Hence the maximum angular velocity can be obtained using $\frac{dx(t_{max})}{dt}/R = \frac{mg\sin(\theta)}{kR}\sqrt{\frac{k}{\frac{I}{R^2}+m}} = 8.7rads^{-1}$

B.

The time period is obviously given by $T = 2\pi\sqrt{\frac{\frac{I}{R^2}+m}{k}} = 1.9s$

III. PROBLEM-3

Let L be the length and w the width of the stick. The moment of Inertia of the stick is given by $\frac{1}{12}M(L^2 + w^2)$ where M is the mass of the stick. Since the width of the stick is much less than the length of the stick we approximate the moment of Inertia as $I = \frac{1}{12}ML^2$. Let m the mass of the puck, h the distance the puck strikes the stick from the centre of mass and v_1 and v_2 be the initial and final velocities of the puck.

A.

Since there are no external torques on the system the angular momentum is conserved. We see that $m(v_1 - v_2)h = I\omega$ where $I = 1/12ML^2$. Therefore $\omega = 1.5rads^{-1}$. The angular displacement after $2s$ is $3rad$. The loss in Kinetic energy is given by $\Delta K = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2 - \frac{1}{2}I\omega^2 = 0.34J$

B.

Now the stick is free to move. Let v_{CM} be the velocity of the centre of mass of the stick. Since linear momentum is now conserved we have $m(v_1 - v_2) = Mv_{CM} = 0.625ms^{-1}$. The loss in energy is now given by $\Delta K = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2 - \frac{1}{2}Mv_{CM}^2 - \frac{1}{2}I\omega^2 = 0.26J$

IV. PROBLEM-4

Let f_p be the force of friction between the plank and the cylinders. Let f_c be the force of friction between the cylinders and the flat surface. Let us choose the direction of F as the positive x direction. Let f_p act on the plank along negative-x direction and act on the cylinder along the positive x direction. Also let f_c act along the negative x direction. Let a_c and a_p be the acceleration of the cylinders and the plank. Let v_c and v_p be the velocities of the cylinder and the plank.

The force equation on the plank leads to $F - 2f_p = Ma_p$. Similarly the balance of forces on the cylinders leads to $f_p - f_c = ma_c$. The torques acting on the cylinders are governed by the equation $(f_c + f_p)R = I\alpha$ where α is the angular acceleration of the cylinders. The condition of no slipping implies $\alpha = a_c/R$. Hence $2f_p = (m + I/R^2)a_c$. Using $I = 1/2mR^2$ we have $f_p = 3ma_c/4$.

A.

Now we see that $v_p = v_c + \omega R$. Differenting with respect to time t we obtain $a_p = a_c + \alpha R = 2a_c$. Hence $a_p = \frac{F}{M+\frac{3}{4}m} = 0.8ms^{-2}$ and $a_c = 0.4ms^{-2}$.

B.

$f_p = 3ma_c/4 = 0.6N$ and $f_c = f_p - ma_c = -0.2N$. We note that f_c is acting along the direction of translational motion.