Solutions-Probem set #7

1)

i)

$$P_{av} = \frac{\Delta E}{\Delta t} = \frac{E_f - E_i}{t_f - t_i} = \frac{\frac{1}{2}mv^2}{t}$$

$$v(t) = \sqrt{\frac{2Pt}{m}} \quad \rightarrow \quad v(t=5) = 24.5\, m/s$$

$$a(t) = \frac{dv(t)}{dt} = \sqrt{\frac{2P}{m}}\frac{1}{2}\frac{1}{\sqrt{t}} \quad \rightarrow \quad a(t=5) = 2.45\, m/s^2$$

ii)

$$x(t) = \int_0^t v(t')dt' = \sqrt{\frac{2P}{m}}\frac{2}{3}t^{3/2} \rightarrow x(t=5) = 83m.$$

i)

To determine the speed of the block-bullet system we use conservation of momentum.

$$p_{initial} = p_{final} \quad m_{bullet}v_{bullet} = (m_{bullet} + m_{block})v_f$$

$$v_f = 5.2\, m/s$$

ii)

We use the conservation of energy to determine how high the block goes on the inclined plane

$$E_{total} = E_{initial} = U_g + W_{friction}$$

The friction dissipates a part of the initial energy in heat.

The friction force is constant, therefore :

$$W_{friction} = F_{friction}\Delta d = F_{friction}\frac{h}{\sin(15)}$$

Also : $F_{friction} = N\mu_k = mg\cos(15)\mu_k$

$$\frac{1}{2}mv^2 = mgh + mg\mu_k\frac{h}{\tan(15)}$$

h= 0.88 m

iii) Again we use the conservation of energy, except that the initial energy is only potential and the final energy will only be kinetic. The amount of energy lost in heat is the same.

$$E_{total} = E_{initial} = K + W_{friction}$$

$$mgh = \frac{1}{2}mv_f^{\,2} + mg\mu_k\frac{h}{\tan(15)}$$

$V_f = 2.8$ m/s

1)

The easiest way to solve this problem is to consider the whole system as two geometrical figures, a uniform square tin with side L, centered at (0,0), and an off-center disk having a negative mass.

The mass of the tin : $M = \rho L^2$

The mass of the "negative" disk: m = $-\rho\pi r^2$

Let's find x_{cm} and y_{cm}

We use $x_{cm} = \dfrac{m_1 x_{cm}^{(1)} + m_2 x_{cm}^{(2)}}{m_1 + m_2} = \dfrac{M(0) + m(.06)}{M + m} = \dfrac{0 + (-\pi r^2)(.06)}{L^2 - \pi r^2} = -4.6x10^{-3}\,m$

Similarly

$$y_{cm} = \frac{m_1 y_{cm}^{(1)} + m_2 y_{cm}^{(2)}}{m_1 + m_2} = \frac{M(0) + m(.04)}{M + m} = \frac{0 + (-\pi r^2)(.04)}{L^2 - \pi r^2} = -3.0x10^{-3}\,m$$

2) The rate of change of momentum is equal to the external force applied. $\vec{F} = \dfrac{d\vec{P}}{dt} = \dfrac{d(m\vec{v})}{dt} = m\dfrac{d\vec{v}}{dt} + \vec{v}\dfrac{dm}{dt} = \vec{v}\dfrac{dm}{dt}$ when v is constant, dv/dt is zero

The force allows the sand to be carried at a constant speed, even if the changing mass on top of the conveyor belt is increasing. Because of the conservation of the momentum, the rate of change in the momentum of the conveyor belt induced by the falling sand is also equal to the rate of change of the falling sand.

a) The mass on the conveyor belt is increasing, its rate of change of momentum is :
v(dm/dt) = 0.750*5 =3.75 N
The sand's rate of change of momentum in the horizontal direction is therefore:
3.75 N (in the forward direction)

b) The force of friction applied by the belt is the only horizontal force acting on the sand in the horizontal direction. To induce this change of momentum on the sand, this force has to equal:
F_f = v(dm/dt) = 3.75 N (in the forward direction)

c) The friction on sand is forward direction, so its reaction on the belt is in negative direction and to keep the belt moving with the same velocity we have to apply equal force in forward direction

F_{ext}= v(dm/dt) = 3.75 N (in the forward direction)

d) The Force is constant and sand moves at a constant velocity. The work done is :

$$W = F \cdot v \cdot t = 2.85\ J$$

e) $K.E. = \frac{1}{2}v^2\left(\frac{dm}{dt}\right)t$

Each second the change in kinetic energy is $\frac{1}{2}v^2\left(\frac{dm}{dt}\right)$=1.41 J

f) Part of the energy provided by the conveyor belt is lost to friction (friction is a non-conservative force) to prevent the sand from slipping backward relative to the belt.

increase in K.E. + work done against friction = work done by external force