Problem set # 5 Solutions

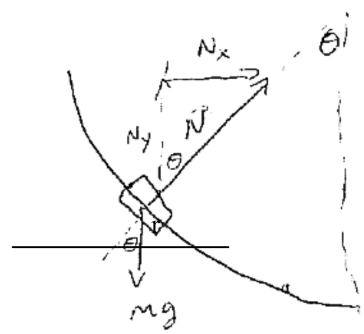
1) The force on Alice (of mass *m*) if she is in contact with the platform is due to gravity and the platform. If Alice loses contact with the platform then only gravity acts downward on her and she falls with acceleration $-g$, taking the up direction to be along positive *y*. But this means, that the platform is also accelerating at least at $-g$. But then we would have $a_{platform} = \frac{d^2y}{dt^2} = -\omega^2 A \sin(\omega t) = -g$. Since the maximum value of the sine function is +1, $\omega = \sqrt{g/A} = 6.3 rad/s$ and the minimum period is $T = \frac{2\pi}{\omega} = 1.0s$.

2 i) Take the up direction to be along $+y$ and the horizontal direction towards the centre of the circle to be $+x$. The two forces acting on the mass are the normal force, $\vec{N}$, directed from the mass towards the centre of the loop, and gravity ($-mg$) which acts along $-y$. Along the vertical (y) direction we have:

$\sum F_y = N\cos\theta - mg = 0$ since the object does not accelerate vertically. The net horizontal force acts as the centripetal force causing rotation on a circle of radius $r = R\sin\theta$ with angular velocity ω. For equilibrium

$$F_x = N\sin\theta = m\omega^2 r$$

$$N = m\omega^2 R$$



Using the result from the *y*-component of the force to eliminate *N* we have

$$\therefore \quad \cos\theta = g/\omega^2 R$$

Note that this angle can only be real if $\omega^2 > g/r$. Hence there is a minimum angular speed associated with establishing an equilibrium position away from the bottom of the loop. For $\omega^2 < g/r$ one would have θ =0.

2 ii) At the top of the swinging motion the total force acting on the water is the result of its weight ($-mg$) and the normal force ($-N$) from the bucket acting (downward) on it. If the water moves in a circle of radius *R*, this total force provides the centripetal force. Hence $-N - mg = -\omega^2 R$. The angular speed will be a minimum if the normal force is zero and the weight alone provides the centripetal force. Hence

$-mg = -\omega^2 R$ or $\omega = \sqrt{g/R} = 2.5$ rad/s

3 i) As we showed in class (or as is done in the text), if we take the positive *y*-direction to be downward Newton's second law along *y* gives,

$$\sum F_y = -bv + mg = m\frac{dv}{dt} \quad \text{or} \quad \frac{-b}{\text{m}}(v - mg/b) = \frac{dv}{dt}$$

let $v' = v - mg/b \qquad \therefore -\frac{b}{m}v' = \frac{dv'}{dt}$

$v'(t) = Ce^{-\frac{b}{m}t}$ and $\text{v} = Ce^{-\frac{b}{m}t} + mg/b$

For the initial condition: $t = 0,\ v = v_0$ $v_0 = C + mg/b$ so $v = (v_0 - mg/b)e^{-\frac{b}{m}t} + mg/b$

Using the values provided with $v_0 = -10$ $\therefore$ $v(t) = (-60e^{-0.2t} + 50)m/s.$

At the top of the trajectory, $v(\text{t}) = 0 = -60e^{-0.2t} + 50 \quad t = 0.93s$

Without drag, $v_\text{i} = -10\ m/s;\ v_\text{f} = 0\ m/s$

$v_f = v_i + gt \quad t = 1.02s$

So the ball reaches the top of its trajectory sooner when there is drag (but doesn't go as high).

3 ii) The distance the ball travels is
$y(t) = \int v(t)dt = +300e^{-0.2t} + 50t + B$ where B is a constant.
Since initially $y = 0$ at $t = 0$ we have B = -300. When the ball returns to its starting position $y = 0$ again. Hence we have to find $t \neq 0$ such that $+300e^{-0.2t} + 50t - 300. = 0$.
Using numerical or graphical techniques we find t = 1.9 s. At this time the ball has a speed of $v(t = 1.9) = (-60e^{-0.2(1.9)} + 50)m/s = 8.8m/s.$

This illustrates how even simple resistive force problems can become numerically intensive!

4 i) Take the up direction to be along positive y. The gravitational force is $-mg\hat{j}$, and the work done by gravity depends only on its initial and final y positions.

$W = -mg(y_f - y_i)$

$y_f - y_i = v_{oy}t - 1/2gt^2 = v_o Sin(45^0)t - 1/2gt^2$

For $t =$ 1 s $\ y_f - y_i = 9.3m \quad \therefore W = -46N$

ii) If we take up to be along positive y, the maximum compression of the spring is (from the solutions to problem set #4) y_{max} = -0.89 m. For the work done by friction (force = +250N) $W = F_{fric}(y_{max} - 0) = (250)(-0.89) = -2.2x10^2 J$. The work done by the spring $W_{spring} = \int_0^{y_{max}}(-ky)dy = -1/2k(y_{max})^2 = \frac{-1}{2}(4000)(-0.89)^2 = -1.6x10^3 J$. For the compound (series) spring with k' = 2.4 kN/m the platform dropped by y_{max} = -1.2 m

$$W_{friction} = F_{fric}(y_{max} - 0) = 250(-1.2) = -3.0x10^3 J$$

$$W_{spring} = -1/2k'(y_{max})^2 = -\frac{1}{2}(2400)(-1.2)^2 = -1.7x10^3 J$$