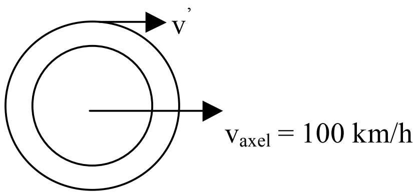
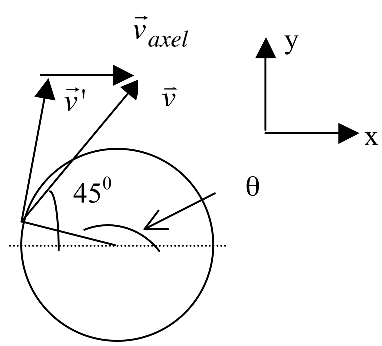
**#1** a) If $\vec{v}_{axel}$ is the velocity of the axel which is moving forward relative to the ground (at 100 km/hr) and $\vec{v}'$ is the velocity of any point on the wheel relative to the axel, then the velocity of any point relative to the ground is $\vec{v} = \vec{v}_{axel} + \vec{v}'$. If $r_{rim}$ is the radius of the inner rim which rolls without slipping we have that for the bottom point on the rim $\vec{v}_{bottom\ of\ rim} = 0 = \vec{v}_{axel} + \vec{v}'$ or if $\vec{v}_{axel} = 100\ km/hr\ \hat{i}$, $\vec{v}' = -100\ km/hr\ \hat{i} = -r_{rim}\omega\hat{\theta}_{bottom}$ where ω is the constant (negative) angular speed. For a point a distance r from the centre, $\vec{v}' = r\,\omega\hat{\theta} = \dfrac{r}{r_{rim}}(100\ km/hr)\hat{\theta}$. Since we are looking for the maximum speed associated with $\vec{v} = \vec{v}_{axel} + \vec{v}'$ and since both components have constant magnitude we will want both vectors pointing in the same direction with a point at the maximum distance, $r_{flange}$, from the centre. We have $\vec{v}' = r_{flange}\omega\hat{\theta} = \dfrac{r_{flange}}{r_{rim}}(100\ km/hr)\hat{\theta}$, and the maximum speed occurs for a point at the top of the wheel (angular position π/2 relative to horizontal) .




Then $v = 100 km/hr + \dfrac{r_{flange}}{r_{rim}} 100 km/hr$ = $2.2 \text{x} 10^2$ km/hr.

b) As indicated above the angular position π/2 relative to horizontal) .

c)

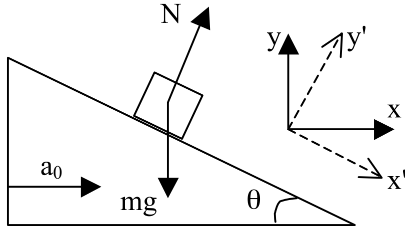



To have the velocity vector make an angle of $+45^0$ with the <u>forward</u> horizontal direction, $v_x = v_y$ with $v_x > 0$. Using the fact that for polar angle *θ*, the unit tangent vector is $\sin\theta\hat{i} - \cos\theta\,\hat{j}$ so that

$v_x = 100\ km/hr + \dfrac{r_{flange}}{r_{rim}} 100\ km/hr \sin\theta$ and $v_y = -\dfrac{r_{flange}}{r_{rim}} 100\ km/hr \cos\theta$

or $\dfrac{1 + 1.17\sin\theta}{-1.17\cos\theta} = 1$. Solving, we find $\theta = 173^0$.

**#2** i)




If $\vec{a}$ is the acceleration of the block in the 'ground" frame (the inertial frame) and $\vec{a}'$ is the acceleration of the block in the fame of the plane, we have (by taking two time derivatives of position vectors in the respective frames) that $\vec{a} = \vec{a}' + \vec{a}_0$ and $m\vec{a} = \vec{F} = m\vec{a}' + m\vec{a}_0$ where $\vec{F}$ are the physical forces acting on the mass, *m,* and $\vec{a}_0$ is the accerlation of plane along the x-direction. Taking components along the plane direction (x') we have that the only component of force is $mg\sin\theta$ and so $mg\sin\theta = ma_{x'} + ma_0\cos\theta$. This gives $a_{x'} = g\sin\theta - a_0\cos\theta$. Similarly, if we take components of the force equation perpendicular to the incline plane, if the block remains in contact with the plane, then it can't accelerate across the plane and $a_{y'} = 0$. The mass can lose contact with the plane if the normal force (acting along y') is zero. Then along y', $N - mg\cos\theta = -mg\cos\theta = ma_{y'} + ma_0\sin\theta$ and $a_{y'} = -mg\cos\theta - ma_0\sin\theta$. The critical angle at which the mass would lose contact with the plane is found by setting $a_{y'}$ to zero or $\tan\theta_{crit} = -\dfrac{g}{a_o}$.

ii) Consider the forces along the plane

$$\sum F_{x'} = mg\sin\theta - f = ma_{x'}$$

$$\sum F_{x'} = \sin(\theta)mg - f = ma_x$$

$\sum F_{y'} = N - \cos(\theta)mg = 0$ block stays in contact with the plane

$a_{x'} > 0$ beginning at $\theta = \pi/7$ if $f_{static} = \sin(\pi/7)mg$

$$\mu_{static} = \frac{f_{static}}{N} = \tan(\pi/7) = 0.48.$$

$x'_f - x'_i = 0.3\ m = \frac{1}{2}a_{x'}t^2$, $a_{x'} = 0.5\ m/s^2$.

$$f_{kinetic} = \sin\theta mg - ma_{x'}$$

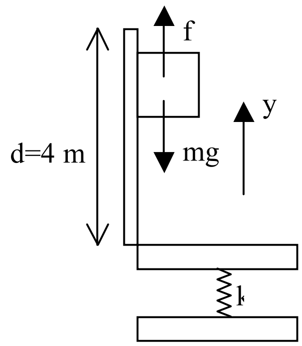
$$\mu_{kinetic} = \frac{f_{kinetic}}{N} = \frac{g\sin(\pi/7) - a_{x'}}{g\cos(\pi/7)} = 0.43$$

Find angle $\theta$ where $a_{x'} = 0$.

$$\sum F_{x'} = \sin(\theta)mg - f_{kinetic} = \sin(\theta)mg - \mu_{kinetic}N = 0$$

$$\theta = \tan^{-1}(\mu_{kinetic}) = 0.40\ rad. = 23°.$$

**#3** i)




Let *f* be the force due to friction. The force on Catherine $f - mg = ma_y$ or $a_y = -5.3 m/s^2$.

$y_i - y_f = -4m = v_{yi}t + \frac{1}{2}a_y t^2 = \frac{1}{2}a_y t^2$. Hence $t$= 1.3 *s* and $v_{yf} = v_{yi} + a_y t = a_y t$= -6.9 *m/s*.

ii) When Catherine strikes the spring it is uncompressed and she has starts it moving at -6.9 *m/s.* The equation of motion of the spring until it reaches maximum compression friction only in the vertical direction) is $m\frac{d^2y}{dt^2} = -ky - mg + F_f = -k(y + mg/k - F_f/k)$ or with $y' = y + mg/k - F_f/k$ we have $m\frac{d^2y'}{dt^2} = -ky'$. The general solution is

$y' = A\cos(\omega t + \phi_i)$ $y = A\cos(\omega t + \phi_i) - mg/k + F_f/k$ where $\omega = \sqrt{k/m} = 8.5$ *rad/s.*

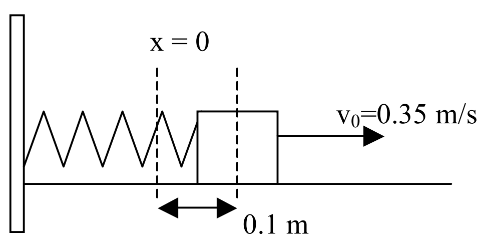
Using $y(t=0) = 0 = A\cos(\phi_i) - mg/k + F_f/k = A\cos(\phi_i) - 3/40$ and

$\frac{dy}{dt}(t=0) = -6.9 = -A\omega\sin(\phi_i)$ we have

$\phi_i = 1.5$ *rad*, A= 1.6 m. The maximum compression occurs when y attains its lowest value (*i.e.*, when $\cos(\omega t + \phi_i) = -1$ for which y= -0.89 *m*.

iii) For two springs connected in series, each spring experiences the total tension (*T*) so the total compression is $\frac{T}{k_1} + \frac{T}{k_2} = \frac{T}{k_1 k_2/(k_1 + k_2)}$. The effective spring constant is therefore $k_s$ =2.4 *kN/m*. Repeating the above calculation with the new spring constant gives $y_{max}$= -1.2 *m*.

**#4**




The mass executes simple harmonic motion of the form $x = A\cos(\omega t + \phi)$. From the initial conditions ($t = 0$) we have $0.1 = A\cos\phi$ [Eq.1] and from $v = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$ we have $v(t=0) = v_0 = 0.35 = -A\omega\sin(\phi)$ [Eq. 2]. Combining Eqs. 1 and 2 we have

$-3.5 = \omega \tan\phi$. Also we have at some later time, t, $v = -0.2 = -A\omega\sin(\omega t + \phi)$ and $x(t) = 0.17 = A\cos(\omega t + \phi)$. Combining the last two equations we have $-0.2 = -\omega\sqrt{A^2 - (0.17)^2}$. Combining the first two equations (1&2) we have $3.5 = \omega\sqrt{A^2 - (0.1)^2}$ . Hence $A$= 0.2 *m* and $\phi$=-π/3. Also ω= 2 *rad/s*. After 1s we have $x$ = 0.12 *m* and $v$ = -0.32 *m/s*.