Solutions Set 3

1. Use: $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$. From the information given

$$\frac{\pi}{3} = \frac{\pi}{8} + \omega_0 + \frac{1}{2}\alpha$$

$$\frac{\pi}{2} = \frac{\pi}{8} + 2\omega_0 + 2\alpha$$

Hence $\boxed{\omega_0 = \frac{11\pi}{48}\ rad/s}$ and $\boxed{\alpha = -\frac{\pi}{24}\ rad/s^2}$

After 10 *s,* the angular position is π/3 *rad* and $|\omega| = |\omega_0 + \alpha(10s)| = 9\pi/48$ *rad/s.*

2 a) The radial and tangential accelerations components are given by, $\vec{a}_t = \alpha R_0 \hat{\theta}$ and $\vec{a}_r = -\omega^2 R_0 \hat{r} = -\alpha^2 t^2 R_0 \hat{r}$ and $\vec{a} = \vec{a}_t + \vec{a}_r$ is the total acceleration, if the rock starts from rest. The linear velocity is $\vec{v} = \omega R_0 \hat{\theta} = \alpha t R_0 \hat{\theta}$. At t = 1 s we have that

$\vec{a} \cdot \vec{v} = av\cos(9\pi/20)$ or

$$\alpha(0.5m)(\alpha(1s)(0.5m)\hat{\theta}\cdot\hat{\theta} = \sqrt{[\alpha^2(1s)^2(0.5m]^2 + [\alpha(0.5m)]^2} \times |\alpha(0.5)(1s)|\cos(9\pi/20)$$

Hence $\alpha = 6.3$ rad s^{-2}. After 4 sec., assuming the ball started at $\theta_0 = 0$,

$\theta = \frac{1}{2}\alpha t^2 = 50.5\ rad$ or $\boxed{\text{8 full revolutions}}$.

b) At time t = 4 s, the angle, ϕ, between the linear velocity and total acceleration is given by $\vec{a}\cdot\vec{v} = av\cos(\phi)$. From $\vec{a} = \vec{a}_t + \vec{a}_r = (6.3s^{-2})(0.5m)\hat{\theta} - (6.3s^{-2})^2(4s)^2(0.5m)\hat{r}$ and $v = (6.3s^{-2})(4s)(0.5m)\hat{\theta}$ we get $\phi \approx \pi/2$.

c) In general if the ball is located at position $\vec{r} = R_0\hat{r}(\theta)$ and it has velocity $\vec{v} = R_0\omega\hat{\theta}(\theta)$ at the time of the break, t_1, the distance from the point where the ball started is $\left|R_0\hat{r}(\theta) + \vec{v}t_1\hat{\theta}(\theta) - R_0\hat{r}(\theta = 0)\right| = \left|\alpha R_0 t_1\hat{\theta}(\theta)\right|$. Setting t_1 = 1 s (flight time) (with θ =8 rev.=0 rad) we find that the rock travels $\boxed{13\ m}$ from the point where it started.

3. Let the positive x-direction be the direction of water flow with $\vec{u} = u\hat{i} = 0.1\hat{i}$ (m/s) so that the y-direction is directly across the river. Justin's velocity relative to the water is $\vec{v}' = v'_x\hat{i} + v'_y\hat{j} = v'\cos\theta'\hat{i} + v'\sin\theta'\hat{j}$ and his velocity relative to the shore is $\vec{v} = v_x\hat{i} + v_y\hat{j} = (v'_x + u)\hat{i} + v_y\hat{j} = v\cos\theta\hat{i} + v\sin\theta\hat{j}$. Given that he arrives on the opposite shore, the x-component of his velocity relative to the shore is zero $v'\cos\theta' + u = v_x = 0$, and since he arrives in 90 sec, $50/v_y = 50/v'\sin\theta' = 90$. Hence $\theta' = \tan^{-1}(-50/9) = 1.75$ rad=100 degrees (he has to point himself upstream slightly relative to the water). This gives *v'*= 0.56 *m/s,* his speed relative to the water.

If Justin ends up 5 m upstream, if the origin is chosen to be his starting point, his final position is x= - 5, y = 50 (in meters). Hence $t = x/(u + v'\cos\theta') = y/v'\sin\theta'$ gives θ'= 1.83 *rad* (105°) and t = $\boxed{92\ s}$. If x= 5, y = 50 θ'= 1.66 rad. (95°) and t =$\boxed{88\ s}$.

4. Define :

- v_b: Carole's burst velocity relative to road
- t_1: time it takes Carole to get to front of pack
- t_2: time it takes Carole to slip to back of pack starting at front

Solve the problem in the frame of the pack of riders. In this frame, when she makes a burst her velocity is $v_b - 9$ and she moves 50 m. When she tires her velocity in the riders' frame is $v_b/2 - 9$ and she moves -100 m. Hence $t_1 + t_2 = 50 = \frac{50}{v_b - 9} + \frac{(-100)}{v_b/2 - 9}$

Solving the quadratic equation for v_b, $v_b = 12$ *m/s*, $t_1 = 17$ *s*.