Solutions: Problem set #1

1. A glance at a map shows that Greenland is more than 10^3 km long but less than 10^3 km across, so a reasonable estimate of its area is 10^6 (km)^2. If all the ice on Greenland melts one has about 10^7 (km)^3 of water released to the oceans. Since the oceans cover 2/3 of the Earth (radius, R_E =7 x 10^3 km), which has a surface area of $4\pi R_E^2$ the rise in sea level is $\dfrac{10^7(km)^3}{\frac{2}{3}4\pi(7x10^3\,km)^2} \approx 25\ m$. This is of the same order of magnitude as the *Globe and Mail*'s number given various approximations (thickness of ice sheet, etc.).

2. i) All terms must have the dimension of length. Hence $[am^2t^3]$ = L. By inspection $[a] = LM^{-2}\,T^{-3}$. For the second term $[bc^3Gv^2] = L$. But $[bc^3Gv^2]=[b][c^3][G][v^2] = [b][L^3T^{-3}][FL^2M^{-2}][L^2T^{-2}]= [b][L^3T^{-3}]\,[MLT^{-2}L^2M^{-2}][\,L^2T^{-2}]= [b]L^8\,M^{-1}T^{-7}$. Hence $[b] = L^{-7}\,MT^7$.

ii) Matching dimensions of the RHS and LHS we have $[period] = T = [L^x][g^y][m^z] = L^{x+y}T^{-2y}M^z$. We have $x + y = 0$, $y = -1/2$, $z = 0$ so that $x = 1/2$. Therefore $T = k\sqrt{L/g}$.

3. i) Let the bridge be at height $x = 0$ and the first rock be dropped at t = 0. Its position is $x_1 = -\frac{1}{2}gt^2$. If the second rock is dropped with speed u, its vertical position after it is dropped is $x_2 = -u(t-1) - \frac{1}{2}g(t-1)^2$. When the rocks hit the river $x_1 = x_2 = -40$. From the position equation for the first rock we get t = $\sqrt{8}$ *s*. Hence from the position equation for the second rock, u = 13 ms^{-1}.

ii) Relative to the slow train the fast train is approaching at a speed of v_1-v_2 and is initially at a separation of $-d$. The fast train must have acceleration, $a < 0$ and with its maximum acceleration (minimum *magnitude* of acceleration) both trains would be moving at same speed (relative speed = 0) when they almost touch. Hence $(v_2 - v_1)^2 - 0^2 = 2a_{\max}(-d)$. For no collision $a < a_{\max} = -\dfrac{(v_2 - v_1)^2}{2d}$.

4. The instantaneous velocity and acceleration are given by $v = \dfrac{dx}{dt} = 0.3(2\pi)\cos 2\pi t$ and $a = \dfrac{dv}{dt} = -0.3(2\pi)^2 \sin 2\pi t$. The speed and acceleration at t = 1 s are $|v(t=1)| = 0.6\pi\, m/s$ =1.9 *m/s* and $a(t=1) = 0$. The amplitude of the periodic motion is 0.3 m and the motion repeats every 1 s (the period). In each period the block moves a distance of 4 times its maximum amplitude or 1.2 *m*. In 4 *s* it moves a distance 4.8 *m*. In the next 0.3 s it is at x= 0.28 m; it went to its maximum displacement and moved back a distance 0.02. Hence the total distance traveled =4.8+0.3+0.02 = 5.1(2) *m*. The average speed of the block in this time interval = distance/time = 5.1/4.3= 1.2 *m/s*. The average velocity is {x(t = 4.3)-x(t = 0)}/4.3 = 0.28/4.3 = 6.5x10^{-2} *m/s*.