Solutions Term test #2: Phy 180

November 16, 2006

1)a)

i)The theorem states that the moment of inertia of a system about any axis is equal to the moment of inertia about a parallel axis passing through the center of mass plus the product of the mass of the system and the square of the distance separating the two axes.

ii) Power is the time rate of change of (mechanical) work done by the system. Efficiency is the percentage of energy expended by the system that can perform mechanical work.

iii) The condition for unstable equilibrium for a particle is that it is located at a local maximum of the potential energy.

iv) An elastic collision is one which preserves kinetic energy while a perfectly inelastic collision is one in which both objects move together afterwards (but not one in which all kinetic energy is lost!).

v) The thrust of a rocket is determined by the product of the rate at which it expels mass in the backward direction and the speed at which mass is expelled relative to the rocket.

b) The work done by an external influence on an object is

$$W = \int \vec{F}\cdot d\vec{r} = \int m\frac{d\vec{v}}{dt}\cdot d\vec{r} = \int m d\vec{v}\cdot\frac{d\vec{r}}{dt} = \int m d\vec{v}\cdot\vec{v} = m\sum_\alpha \int v_\alpha dv_\alpha = \frac{m}{2}\sum_\alpha v_\alpha^2\Big|_{initial}^{final}$$

$\frac{m}{2}(v_f^2 - v_i^2) = \Delta K$. Note: because $d\vec{v}$ is not necessarily parallel to $\vec{v}$ one <u>cannot</u> use as an intermediate step $d\vec{v}\cdot\vec{v} = v dv$ as Serway does for 1-D.

For a conservative force $W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}\cdot d\vec{r} = -U(\vec{r}_f) + U(\vec{r}_i)$ (independent of the path) since the work done only depends on a function of the endpoints. Hence if we have two different pathways to get from the initial to the final position $W = \int_{pathA} \vec{F}\cdot d\vec{r} = \int_{pathB} \vec{F}\cdot d\vec{r}$ so that for a closed path formed from the two different paths

$$W_{closed} = \int_{pathA} \vec{F}\cdot d\vec{r} + \int_{pathB} \vec{F}\cdot d(-\vec{r}) = \int_{pathA} \vec{F}\cdot d\vec{r} - \int_{pathB} \vec{F}\cdot d\vec{r} = 0$$

2) i) The kinetic energy of the block after it is released by the spring is ½(0.5kg)$(12)^2$= 36 J. This is equal to $1/2kx^2$ the initial stored energy of the spring. For k = 450 N/m, we have x=2/5 m.

ii) By the work kinetic energy theorem $W = -\Delta U + F_f d = K_f - K_i = 1/2(0.5kg)v^2 - 36J$. For a height corresponding to angle θ (measured with respect to a downward pointing radial vector)

$-\Delta U + F_f d = mg(R - R\cos\theta) + F_f\theta$.

For $\theta = \pi$ (block at maximum height) we have $-mg(R-R\cos\theta)+F_f\theta = -0.5kg\ (9.8)(1.)(2) - 7(1.)(3.14) = 9.8 - 22 = -31.8J$.

Therefore $1/2(0.5kg)v^2 = 4.2$ J and since $v^2 = 16.8\ m^2/s^2 > gR = 9.8\ m^2/s^2$ (as required for the minimum centrifugal force) the object is capable or remaining on a circle of radius 1 m (iii).

3. i) Take the origin to be halfway between the masses which both lie on an x-axis. The centre of mass is located at a point where $x_{cm} = [m_1 x_1 + m_3 x_3]/(m_1+m_3) = [(1)(-0.5) + 3(0.5)\ m]/4 = ¼$ m. The moment of inertia about the centre of mass is $I_{cm} = 1(3/4)^2 + 3(1/4)^2 = 3/4\ kgm^2$.

ii) The torque about the centre of mass is $\vec{\tau}_{cm} = \vec{r} \times \vec{F} = r_3 F \sin\theta\ \hat{k} = (1/4)(10)\sin 60^0\ \hat{k} = \frac{5}{4}\sqrt{3}\ \hat{k}$ Nm where $\hat{k}$ is an axis out of the page. The angular acceleration is $\alpha = \tau_{cm}/I_{cm} = \frac{5}{4}\sqrt{3}\ \hat{k}\ /(3/4) = \frac{5\sqrt{3}}{3}\ \hat{k}\ s^{-2}$

iii) The linear acceleration of the center of mass is $\vec{F}/(m_1+m_2) = \vec{a}_{cm} = 2.5\ ms^{-2}$ in the direction of the force. If we call the "right-left" axis *x*, and the "up-down" axis *y*, $\vec{a}_{cm} = 2.5(\cos 60^o\,\hat{x} + \sin 60^o\,\hat{y}) = \frac{5}{4}\hat{x} + \frac{5\sqrt{3}}{4}\hat{y}\ ms^{-2}$. The tangential (along $\hat{y}$) acceleration of the 1 kg mass relative to the center of mass is $-\alpha r_1 = -\frac{5\sqrt{3}}{3}s^{-2}(3/4)m = -\frac{5\sqrt{3}}{4}\hat{y}\ ms^{-2}$ so that the linear acceleration of the 1 kg mass relative to the table is $\vec{a}_{cm} - \alpha r_1\hat{y} = \frac{5}{4}\hat{x}$ ms^{-2}. For the 3 kg mass the tangential ($\hat{y}$) acceleration is $\alpha r_3 = \frac{5\sqrt{3}}{3}s^{-2}(1/4)m = \frac{5\sqrt{3}}{12}\hat{y}$ ms^{-2} so that the linear acceleration of the 3 kg mass relative to the table is $\vec{a}_{cm} + \alpha r_3\hat{y} = \frac{5}{4}\hat{x} + (\frac{5\sqrt{3}}{4} + \frac{5\sqrt{3}}{12})\hat{y}\ ms^{-2}$.

iv) The linear acceleration of any point on the barbell relative to the centre of mass is along the *y* direction. The acceleration of the centre of mass has a horizontal component. Hence there can be no point that has total acceleration relative to the table of zero.

v) The kinetic energy of rotation about the cm is $\frac{1}{2}I_{cm}\omega^2$ while the kinetic energy of translation is $\frac{1}{2}Mv_{cm}^2$. Therefore the ratio we seek is (for t near zero):

$$\frac{\frac{1}{2}I_{cm}\omega^2+\frac{1}{2}Mv_{cm}^2}{\frac{1}{2}I_{cm}\omega^2}=1+\frac{Mv_{cm}^2}{I_{cm}\omega^2}=1+\frac{M(a_{cm}t)^2}{I(\alpha_{cm}t)^2}=1+\frac{4}{3/4}\left(\frac{2.5}{5/\sqrt{3}}\right)^2=5.$$