Solutions
Term Test #1

1.
i) A field force is a fundamental force whose vector value is determine by position relative to the source. A contact force is a phenomenological force whose value is determined by how two objects interact with each other based of physical contact.
ii) Linear velocity is the rate of rate of change of the position vector. Angular velocity is the rate of change of the angular position.
iii) An inertial frame of reference is an idealization whereby an object at rest in such a frame experiences no net forces.
iv) A Galilean transformation relates the kinematic quantities (i.e., position, velocity, etc.) in one inertial frame of reference to those of another frame moving at constant relative velocity.
v) The resonance frequency of a simple harmonic oscillator is the only frequency that a system that obeys Hooke's law will oscillate with if no other forces (e.g. damping, forcing) are applied.

b) Taking down to be the positive direction, the equation of motion is $m\frac{dv}{dt} = mg - bv$ or defining $v' = v - mg/b$ we have $m\frac{dv'}{dt} = -bv'$. The general solution is $v' = Ae^{-bt/m}$ so $v = mg/b + Ae^{-bt/m}$. If the object starts from rest ($v = 0$) at t = 0, then $A = -mg/b$ and $v = \frac{mg}{b}(1 - e^{-bt/m})$. The terminal speed ($t = \infty$) is *mg/b*. If the object starts with speed v_0. we have $v_0 = mg/b + Ae^{-b(0)/m}$ or $A = v_o - mg/b$. Then $v = \frac{mg}{b} + (v_0 - mg/b)e^{-bt/m}$.

2.
i) $\omega = \omega_i + \alpha t$. At 2 sec, ω = 0+4(2)= 8 rad/s.
ii) The tangential speed is $R\omega$= (1)(8)= 8m/s. The acceleration (which can be derived from the basic definition of position vector on a circle) is $R\alpha\hat{\theta} - R\omega^2\hat{r} = (1)(4)\hat{\theta} - (1)8^2\hat{r} = 4\hat{\theta} - 64\hat{r}\ (m/s^2)$. Direction of unit vector is specified by angular position.
iii) The angular position is given by $\theta = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 =$ $\theta = 1 + \frac{1}{2}(4)(4) = 9\ rad$.

3)
i) The force along the incline is given by the component of the weight along the incline and the frictional force, or $F = -mg\,sin\,\theta - \mu mg\,cos\,\theta = -4.9 - 2.4\sqrt{3} = -9.1N$. The acceleration for the 1 kg block is therefore -9.1 m/s^2.

ii) At top of incline L=$2/cos\,30^0 = 2.3\,m$, and $v_{top}^2 - v_i^2 = 2aL$. (Or you can use the hint at end of test, find time and then find v_{top} from $v = v_{top} + at$.) Either way v_{top} = 7.6 m/s.

iii) The time to reach the top of incline is found using hint, if necessary.
$L = 2.3 = L_0 + v_i t + \frac{1}{2}at^2 = 0 + 10t + 4.55t^2$, This gives 0.2 s. At the moment the sled leaves the incline it has a vertical component of speed $v_{top}\,sin\,30^0$ = 3.8 m/s. The time to reach maximum height is found from $v_y = 0 = v_{top}\,sin\,30^0 - gt$ or t = 0.4 s after reaching top of incline. Therefore the time to reach the maximum height from launch is 0.6 s.

iv) The height above the incline is $h = v_{top}\,sin\,30^0\,t - \frac{1}{2}gt^2$
$= 3.8(0.4) - \frac{1}{2}9.8(0.4)^2 = 0.7\,m$. The horizontal distance from the top is
$L = v_{top}\,cos\,30^0\,t = 6.6(0.39) = 2.6\,m$. Therefore the distance from the top is
$\sqrt{2.6^2 + 0.7^2}$ =2.7 m.

v) When object is back at original vertical height, relative to top of incline
$y = -1. = (v_{top})\,sin\,30^0\,t - \frac{1}{2}gt^2$. This gives time of 1 s and vertical component of speed = $(v_{top})\,sin\,30^0 - gt = 3.8 - 9.8(1)$ = -6 m/s. Horizontal component is (always) 6.6 m/s.
Therefore speed is $\sqrt{6.6^2 + 6^2}$ =8.9 m/s