PROBLEM SET 6: SOLUTIONS

NIR ROTENBERG

Problem 1: 7.58 Serway

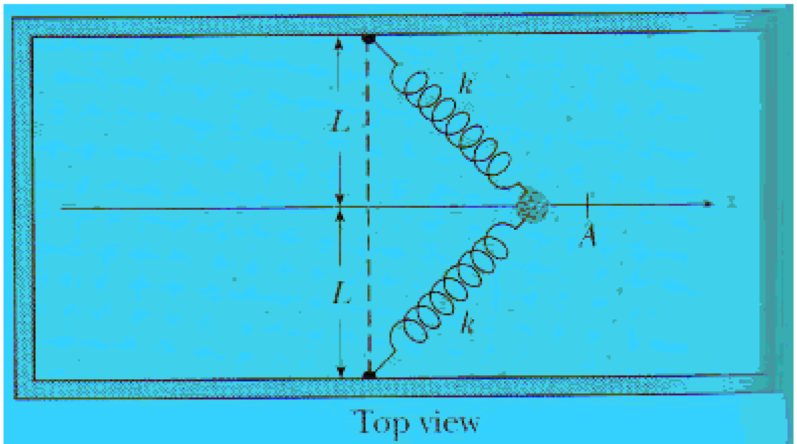

Figure P7.58

(a) If we displace the object by a distance x along the perpendicular direction, then the total length of each spring is $\sqrt{L^2+x^2}$ and the force exerted by EACH spring on the object is $-k\left(\sqrt{L^2+x^2}-L\right)$ directed along the spring to the fixed end. When the two springs are considered together the y-components cancel, while the x-components add. Using the fact that the cosine of the angle between the spring and the x-axis is $x/\sqrt{L^2+x^2}$ from the diagram, the x-component of the force from BOTH springs is

$$F_x = -2k(\sqrt{L^2+x^2}-L)\frac{x}{\sqrt{L^2+x^2}} = -2kx(1-\frac{L}{\sqrt{L^2+x^2}})$$

(b) $W = \int F_x dx = -2k\int_A^0 x dx + kL\int_A^0 2x(x^2+L^2)^{-1/2}dx$. Using (although you may recognize the integrand in the second part as an exact differential) the table for indefinite integrals from Table B.5 of Serway

$$W = \int F_x dx = -2k\frac{x^2}{2}\Bigg|_A^0 + kL\frac{\left(x^2+L^2\right)^{1/2}}{1/2}\Bigg|_A^0$$

$$= \boxed{W = kA^2 + 2kL^2 - 2kL\sqrt{L^2+A^2}}$$

Problem 2: 7.66 Serway

Note that the cylinder has front surface area A, and moves with velocity $v = \frac{\Delta x}{\Delta t}$. Now, recall that power and work are related by

$$W = \int P dt$$

For constant velocity,

$$P\Delta t = W = \Delta K = \frac{1}{2}\Delta m v^2$$

Now, we are asked to express the power in terms of the density of the air pushed. Since density is mass divided by volume,

$$\rho = \frac{\Delta m}{\Delta V} = \frac{\Delta m}{A\Delta x} = \frac{\Delta m}{Av\Delta t}$$

$$\therefore \Delta m = \rho A v \Delta t$$

Inserting this into the equation for the power, and noting that since both sides contain Δt the total power lost to air resistance is:

$$\boxed{P = \frac{1}{2}\rho A v^3}$$

The relation between work and force is:

$$W = \int F dx = F\Delta x$$

where the last line is true if a constant force is applied, as in our case. Inserting this into the power equation,

$$P\Delta t = F\Delta x$$

$$P = \frac{\Delta x}{\Delta t}F$$

$$F = \frac{P}{v}$$

Therefore, the resistive force acting on the car is:

$$\boxed{\frac{1}{2}\rho A v^2}$$

Our model gives a result that is identical to the empirical equation except that the Drag coefficient is, $D = 1$. This is due to the assumption that the air pushes an ever-growing amount of air in front of it. In reality, the air will slip around the car and $D < 1$ (Hope all you future aerospace engineers got this one).

Problem 3: 8.70 Serway

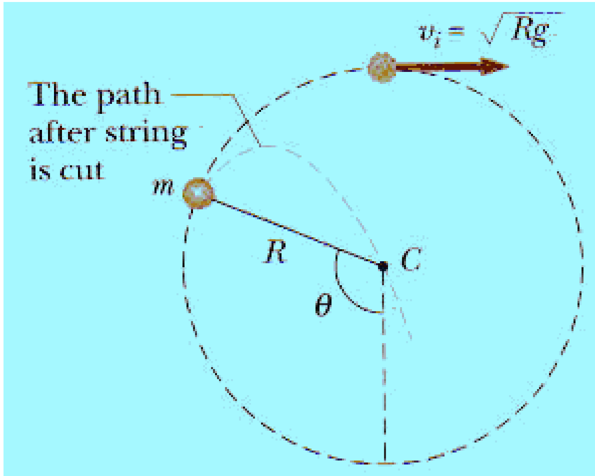

Figure P8.70

If we define the zero height level to be at the centre of the circle, the angle the string makes with this horizontal is $\phi = \theta - 90$ degrees. Therefore

$$x = R\cos\phi \quad v_x = v_f \sin\phi$$

$$y = R\sin\phi \quad v_y = v_f \cos\phi$$

where *x* and *y* are the horizontal and vertical displacements from the centre of the circle at the point when the string is cut, and v_x and v_y are the components of the velocity of the velocity, v_f, at this instant.

(1) Energy balance:

$$E_i = E_f$$

$$\frac{1}{2}mv_i^2 + mgR = \frac{1}{2}mv_f^2 + mgy$$

$$\frac{1}{2}mgR + mgR = \frac{1}{2}mv_f^2 + mgR\sin\phi$$

$$\frac{1}{2}mv_f^2 = \frac{3}{2}mgR - mgR\sin\phi$$

$$v_f^2 = gR(3 - 2\sin\phi)$$

(2) Condition: ball must pass through centre:

1. in the x-direction

$$x = v_x t \Rightarrow R\cos\phi = v_f \sin\phi t \Rightarrow t = \frac{R\cos\phi}{v_f \sin\phi}$$

2. in the y-direction

$$y + v_y t - \frac{1}{2} g t^2 = 0 \Rightarrow R\sin\phi + v_f \cos\phi t - \frac{1}{2} g t^2 = 0$$

3. Substituting t from x into y

$$R\sin\phi + v_f \cos\phi \frac{R\cos\phi}{v_f \sin\phi} - \frac{1}{2} g \left(\frac{R\cos\phi}{v_f \sin\phi}\right)^2 = 0$$

$$\frac{R}{\sin\phi} - \frac{1}{2} g \left(\frac{R\cos\phi}{v_f \sin\phi}\right)^2 = 0$$

$$\therefore v_f^2 = \frac{1}{2} g R \frac{\cos^2\phi}{\sin\phi}$$

By multiplying the results of both (1) and (2) by $\sin\phi$ and comparing the two we see that, after canceling common terms

$$(3 - 2\sin\phi)\sin\phi = \frac{1}{2}\cos^2\phi = \frac{1}{2}\left(1 - \sin^2\phi\right)$$

$$3\sin^2\phi - 6\sin\phi + 1 = 0$$

$$\sin\phi = 1.82, 0.184$$

Since the first solution is not possible, we find that $\phi = 10.6^\circ$. Recalling the original figure, the required angle at which the string should be cut so that the ball passes through the centre of the circle is $\boxed{\theta = 101^\circ}$.

Problem 4: 8.72 Serway

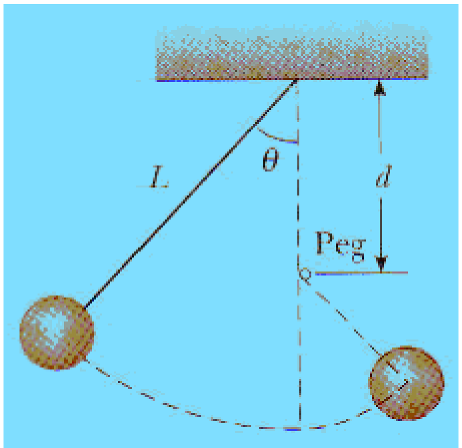

Figure P8.72

(a) Energy is conserved in the swing of the pendulum, and the stationary peg does no work. So the ball's speed does not change when the string hits or leave the peg, and the ball swings equally high on both sides.

(b) The pendulum is started at $\theta = 90^\circ$ and we want it to complete an entire circle about the peg. First we note:

- initial height: $h_i = L$
- radius of circle about peg: $R = L - d$
- height at apex of circle: $h_f = 2R = 2(L-d)$

For the pendulum to circle the peg the velocity at which its tip travels defines the centripetal acceleration. If the tip at the top of its swing is just on the verge of not being on a circle then the centripetal force is provided by gravity alone, with no tension in the string. Therefore

$$a_c = -\frac{v^2}{R} \geq -g$$

Since we are looking for the *minimum* required velocity, we need only solve the equality. Thus, at the apex of the circle about the peg we require

$$v^2 = g(L-d)$$

The energy balance between the start of the swing and the top of the circle is then,

$$mgL + 0 = 2mg\left(L-d\right) + \frac{1}{2}mv^2$$

$$mgL = 2mg\left(L-d\right) + \frac{1}{2}mg\left(L-d\right)$$

$$\frac{5}{2}d = \frac{3}{2}L$$

Therefore: $$\boxed{d = \frac{3}{5}L}$$