Phy180 Problem Set #3 Solutions

1. Let v_m be the speed of Mirna and the caravan.
   Let v_c be Charlie's speed. (The same in both directions.)
   Let t_1 be the time it takes Charlie to get to Mirna and t_2 be the time it takes him to get back.
   Let t be the total time for the trip, so that $t = t_1 + t_2$.

The caravan and Mirna both travel 2.0km in the total time:

$$\Delta d = v_m t = v_m t_1 + v_m t_2 = 2.0\text{km} \quad \text{or} \quad t = \frac{2.0\text{km}}{v_m}$$

With respect to the caravan, Charlie travels 1.5km to Mirna and 1.5km back to the end of the line. His speed with respect to the caravan ($v_c - v_m$) on the way there and ($-v_c - v_m$) on the way back, or ($v_c + v_m$) if we take the absolute value. So for both parts of his trip we have:

$$\begin{aligned} 1.5\text{km} &= (v_c - v_m)\, t_1 \\ 1.5\text{km} &= (v_c + v_m)\, t_2 \end{aligned} \quad \text{or} \quad \begin{aligned} t_1 &= \frac{1.5\text{km}}{(v_c - v_m)} \\ t_2 &= \frac{1.5\text{km}}{(v_c + v_m)} \end{aligned}$$

We can solve for the ratio of the speeds using the fact that $t = t_1 + t_2$: (for simplicity I'll drop the kms)

$$t = t_1 + t_2$$
$$\frac{2}{v_m} = \frac{1.5}{(v_c - v_m)} + \frac{1.5}{(v_c + v_m)}$$
$$0 = 2v_c^2 - 3v_m v_c - 2v_m^2$$
$$0 = 2\left(\frac{v_c}{v_m}\right)^2 - 3\left(\frac{v_c}{v_m}\right) - 2$$

Using the quadratic formula:

$$\left(\frac{v_c}{v_m}\right) = 2, \frac{-1}{2}$$

For Charlie to ever catch up with Mirna, $v_c > v_m$ (and also since speeds are positive), the solution must be $\left(\frac{v_c}{v_m}\right) = 2$.

We're looking for the total distance Charlie travels:

$$D = v_c t$$
$$D = v_c t \times \frac{v_m}{v_m}$$
$$D = \left(\frac{v_c}{v_m}\right) v_m t$$

And $v_m t$ is the total distance traveled by Mirna and the caravan, or 2.0km:

$$D = \left(\frac{v_c}{v_m}\right)(2.0\text{km})$$
$$D = (2)(2.0\text{km})$$
$$D = 4.0\,km$$

2. Given: $\omega_0 = 0\,rad/s$, $\omega_f = 3\,rev/s$, $t = 3\text{s}$

(i) What is the angular acceleration?

First find ω_f in rad/s:

$$\omega_f = 3\,rev/s \times \frac{2\pi\,rad}{1\,rev}$$

$$\omega_f = 6\pi\,rad/s$$

We know:

$$\omega_f = \omega_i + \alpha t$$

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

$$\alpha = \frac{6\pi\,rad/s - 0}{3\text{s}}$$

$$\alpha = 2\pi\,rad/s^2 = 6.3\,rad/s^2$$

(ii) How many revolutions in the last second?

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\Delta\theta = (\omega_f - \alpha t)t + \frac{1}{2}\alpha t^2$$

$$\Delta\theta = \omega_f t - \alpha t^2 + \frac{1}{2}\alpha t^2$$

$$\Delta\theta = \omega_f t - \frac{1}{2}\alpha t^2$$

$$\Delta\theta = (6\pi\,rad/s)(1\text{s}) - \frac{1}{2}(2\pi\,rad/s^2)(1\text{s})^2$$

$$\Delta\theta = 5\pi\,rad$$

In revolutions:

$$\Delta\theta = 5\pi\,rad \times \frac{1\,rev}{2\pi\,rad}$$

$$\Delta\theta = 2.5\,rev$$

(iii) What is the angle between the radius and linear acceleration vectors at t=1s.

After 1s one tooth of the blade has moved: (assume the tooth starts at 0 rad)

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$\theta_f = (0\,rad) + (0\,rad/s)(1\text{s}) + \frac{1}{2}(2\pi\,rad/s^2)(1\text{s})^2$$

$$\theta_f = \pi\,rad$$

So after one second, the tooth has traveled half a rotation, and the radius vector is:

The total linear acceleration is given by $\vec{a}=\vec{a}_t+\vec{a}_c$, where $\vec{a}_t$ is the tangential acceleration and $\vec{a}_c$ is the centripetal acceleration.

First find $\vec{a}_t$:

$a_t = r\alpha$

$a_t=(0.75\,m)(2\pi\, rad/s^2)$

$a_t=4.7\,m/s^2$

$\vec{a}_t=4.7\,m/s^2\hat{\theta}$

Next find $\vec{a}_c$:

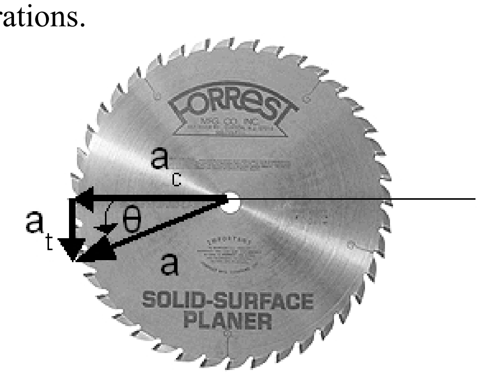
$a_c=r\omega^2$

$a_c=(0.75\,m)(2\pi\, rad/s)^2$

$a_c=29.6\,m/s^2$

$\vec{a}_c=29.6\,m/s^2\,\hat{r}$

The angle between the radius vector and the total linear acceleration is the same as between the tangential and centripetal accelerations.



$$\tan\theta=\frac{a_t}{a_r}$$

$$\tan\theta=\frac{4.7\,m/s^2}{29.6\,m/s^s}$$

$\theta=0.15\,rad=9^o$

(iv) What is the velocity of the sawdust just as it flies off the wheel?

The velocity of the sawdust will be the tangential velocity of the wheel at that point.

$v=r\omega$

$v=(0.75\,m)(6\pi\, rad/s)$

$v=14.1\,m/s$

$\vec{v}=14.1\,ms/\hat{\theta}$

$$R_{wheel} = 65\,cm = 0.65\,m$$

3. Given: $v_{CM} = 20\,km/h = 5.56\,m/s$

$$R_r = R_{wheel}/2 = 0.325\,m$$

Where R_r is the position of the reflector, located half way between the rim and the axle.

(i) Find the ω of the reflector.

$$v_{CM} = R_{wheel}\,\omega$$

$$\omega = \frac{v_{CM}}{R_{wheel}}$$

$$\omega = \frac{(5.56\text{m}/s)}{(0.65\,m)}$$

$$\omega = 8.6\,rad/s$$

(ii) Find the magnitude of the linear acceleration.

The magnitude of the linear acceleration is given by $a = \sqrt{a_t^2 + a_c^2}$. First find a_t:

$$a_t = R_r\,\alpha$$

$$a_t = (0.325\,m)(0\,rad/s)$$

α is 0 since the wheel is moving at a constant speed and hence constant angular speed.

Next find a_c:

$$a_c = R_r\,\omega^2$$

$$a_c = (0.325\text{m})(8.6\text{rad}/s)^2$$

$$a_c = 24\,m/s^2$$

Since a_c is the only component of the acceleration, the magnitude of the total linear acceleration is 24m/s^2.

(iii) What is the maximum speed of the reflector relative to the ground?

The maximum speed of the reflector will be when the reflector is at the top of the wheel. (Refer to Figure 10.29 in Serway.) At that point the speed is given by:

$$v_{max} = v_{CM} + R_r\,\omega$$

$$v_{max} = (5.56\text{m}/s^2) + (0.325\text{m})(8.6\text{rad}/s)$$

$$v_{max} = 8.4\,m/s$$

4. Given $r = b\theta \rightarrow \vec{r} = b\theta\,\hat{r}$
$\omega = 1\,rad/s$
$b = 1\text{m}$

At t=1s, Zhao will be:

$$\theta_f = \theta_i + \omega t$$
$$\theta_f = (1\,rad) + (1\,rad/s)(1\text{s})$$
$$\theta_f = 2\,rad$$

His linear speed will be:

$$\vec{v} = \frac{d\vec{r}}{dt}$$
$$\vec{v} = \left(\frac{dr}{dt}\right)\hat{r} + \left(r\frac{d\theta}{dt}\right)\hat{\theta}$$
$$\vec{v} = (b\omega)\hat{r} + (b\theta\omega)\hat{\theta}$$
$$\vec{v} = (1\text{m})(1\text{rad}/s)\hat{r} + (1\text{m})(2\text{rad})(1\text{rad}/s)\hat{\theta}$$
$$\vec{v} = (1\text{m}/s)\hat{r} + (2\text{m}/s)\hat{\theta}$$

His linear acceleration will be:

$$\vec{a} = \frac{d\vec{v}}{dt}$$
$$\vec{a} = \left(\frac{d^2r}{dt^2} - r\frac{d\theta}{dt}^2\right)\hat{r} + \left(\frac{d^2\theta}{dt^2}r + 2\frac{dr}{dt}\frac{d\theta}{dt}\right)\hat{\theta}$$
$$\vec{a} = \left(-\omega^2 b\theta\right)\hat{r} + \left(2b\omega^2\right)\hat{\theta}$$
$$\vec{a} = -(1\text{rad}/s)^2(1\text{m})(2\text{rad})\hat{r} + 2(1\text{m})(1\text{rad}/s)^2\hat{\theta}$$
$$\vec{a} = (-2\text{m}/s^2)\hat{r} + (2\,m/s^2)\hat{\theta}$$