Problem Set # 2:

"Success seldom comes to those who dream of it, to fewer still who don't." -Anonymous

- 1) i) Peter knows that a conic section (e.g., parabola, ellipse, hyperbola) can be represented in polar co-ordinates by the equation $r = L/(1 e\cos\theta)$ where e and L are constants (the *eccentricity* and *semilatus rectum*, respectively). For what values of e does he obtain a circle? For what values of e does he obtain an ellipse?
 - ii) Represent the ellipse in Cartesian co-ordinates and find its semi-major and semiminor axes in terms of L and e.
 - iii) Newton showed that an object moving in the Sun's gravitational field has the sun at one focus of a conic section. Rachel read that one of the minor reasons Pluto was demoted from planetary status in August, 2006 is that its orbit is too eccentric. If Pluto's *apogee* (furthest distance from sun) is 7.4×10^9 km and its *perigee* (closest distance to the sun) is 4.4×10^9 km, what is the eccentricity of its orbit? (For Earth e = 0.017 and its average distance from the sun is 1.5×10^8 km.)
- 2) With the sun directly overhead, a fly is moving outdoors above a tilted table defined by the plane x + y + 10z = 5, where z is the vertical axis and x, y are horizontal axes. If the fly's path is $\vec{r} = (1 + t^2)\hat{i} + (-4t + 3)\hat{j} + (1 - t^2)\hat{k}$ until it lands, when does it land on the table if it begins flying at t = 0? Is the velocity of the fly ever perpendicular to the table? What is the speed of the fly's shadow along the table at any time t before it lands?
- 3) Daniel is competing in the 90 m ski jump event at the 2010 Olympics in Vancouver. He is currently in second place with a first jump of 95 m and needs a second jump of 100 m to take the Gold medal. Assume the landing hill makes an angle of -60° with respect to the horizontal and begins at the edge of the jump; distances are measured along the hill. By how much (in m/s) must he increase his take-off speed if his take-off velocity remains at 10° above the horizontal?
- 4) Kitty is canoeing upstream on the Black river near Orillia. When she passes under a low bridge her hat falls into the water. She continues paddling for 15 minutes before she notices her hat is missing. She immediately turns around and, paddling at the same speed relative to the water as before, she retrieves her hat 1 km from the bridge. What is the speed of the river relative to its banks? In a separate trip Kitty wants to cross the 100 m wide river. If she paddles at 6 km/hr relative to the water and points her canoe at 30° (as measured by someone on the bank) relative to the river's flow, how long does it take her to cross the river? How far downstream does she reach the other side?