PHYSICS 180

Problem set #1 — Solutions

1) A small glass of water is approximately 5 cm tall and so water evaporates at about 1/2 cm/day on average from an open surface. About 2/3 of the Earth's surface is covered by water and so in one year, the height/depth of water evaporated from the Earth's surface in one year is approximately 2/3 (365 days) 1/2 cm/day = 120 cm or ~ 1 m. Since the rate of evaporation must balance the rate of precipitation, this number is the same as the (average) precipitation on the Earth. (The annual precipitation in Toronto is ~60 cm.)

2) The maxima and minima of the function can be found from
$\frac{dx}{dt} = (5 - t - 3t^2)(-1/2)e^{-t/2} + (-1 - 6t)e^{-t/2} = 0$, giving t = (11±√205)/6. The only t > 0 root is therefore 4.2 s. At this time the object is at x=-6.4 m, or its distance from the hole is 6.4 m. Since 6.4 >5 (the distance at t=0) then 6.4 must be the maximum distance. The displacement in the [1,2] time interval is [x(2)-x(1)] = (5-2-12)e^{-1} –(5-1-3)$e^{-1/2}$ = -9e^{-1}-$e^{-1/2}$ = -3.3-0.6 = -3.9 m. Therefore the average velocity is [x(2)-x(1)] (2-1) = -3.9 m/s. Since there is no reversal of motion in this time interval (this only occurs at 4.2 s), the distance moved is 3.9 m and the average speed is 3.9 m/s. At time "t" the velocity is
$\frac{dx}{dt} = (\frac{-7}{2} - \frac{11}{2}t + \frac{3}{2}t^2)e^{-t/2}$. At t = 1/4 s this has the value -4.2 m/s and so the instantaneous speed here is 4.2 m/s.

3) i) For $F = \frac{Cm_1m_2}{r^2}$, taking dimensions of both sides gives $[F] = MLT^{-2} = [C]M^2L^{-2}$. If $[C] = M^aL^bT^c$, then equating exponents, $a+2=1$, b-2 =1, c+0 =-2. Therefore $[C] = M^{-1}L^3T^{-2}$.

i) All terms must have the dimensions of force or $[F] = MLT^{-2}$ For the first term on the right hand side , if $[A] = M^aL^bT^c$, then $\left[AV^{-3}(\frac{dm}{dt})\right] = M^aL^bT^cL^{-9}MT^{-1} = M^{a+1}L^{b-9}T^{c-1}$

Therefore a=0, b=10, c=-1 or $[C] = L^{10}T^{-1}$. For the second term on the right hand side, if $[B] = ML^2T^{-2}$, then we must have $\left[BT^{-1}L^nT^{-n}\right] = ML^{-2}T^2T^{-1}L^nT^{-n} = MLT^{-2}$ or n=3.

4) 1000 km/hr = 27.8m/s. Deceleration is $a = (v_f - v_i)/\Delta t$ = -27.8 m/s/1.8s= -15.4 m/s^2
Therefore stopping distance is $d_s = \left(v_f^2 - v_i^2\right)/2a$ = -(27.8)^2/2/(-15.4) = 25.1m. Total distance he travels is $v_i(0.4s) + d_s$ = 27.8m/s*0.4s+25.1m= 36.2m which is >30 m, so he would hit the box. To avoid hitting the box he must travel < (30.-25.1)m = 4.9 m during his reaction time and so he would have to react in no more than 4.9/27.8 s or < 0.18s.