Solution to PHY180F Term Test 2 2005

Question 1

(a) Angular momentum is conserved.

Angular Momentum Before = Angular Momentum After

$$\mathbf{L_i}(\text{Mass}) = \mathbf{L_f}(\text{Mass}) + \mathbf{L_f}(\text{Rod})$$

$$\frac{\mathbf{d}}{2}\mathbf{m}\,\mathbf{v} = \mathbf{m}\left(\frac{\mathbf{d}}{2}\right)^2 \boldsymbol{\omega} + \frac{1}{12}\mathbf{M}\,\mathbf{d}^2\boldsymbol{\omega}$$

$$\frac{4}{2}\,\text{x}\,\frac{1}{2}\,\text{x}\,2 = \frac{1}{2}\,\text{x}\left(\frac{4}{2}\right)^2 \boldsymbol{\omega} + \frac{1}{12}\,\text{x}\,5\,\text{x}\,16\,\boldsymbol{\omega}$$

$$2 = 2\,\boldsymbol{\omega} + \frac{20}{3}\boldsymbol{\omega}$$

$$2 = \frac{26}{3}\boldsymbol{\omega}$$

$$\boldsymbol{\omega} = \frac{6}{26} = 0.23\,\text{rad/s}$$

(b) Initial Energy = $\frac{1}{2}\mathbf{m}\,\mathbf{v}^2 = \frac{1}{2}\,\text{x}\,\frac{1}{2}\,\text{x}\,4 = 1.0\text{ J.}$

Final Energy $= \frac{1}{2}\mathbf{I}_{\text{Mass}}\,\boldsymbol{\omega}^2 + \frac{1}{2}\mathbf{I}_{\text{Rod}}\,\boldsymbol{\omega}^2$

$$= \frac{1}{2}\mathbf{m}\frac{\mathbf{d}^2}{4}\boldsymbol{\omega}^2 + \frac{1}{2}\,\text{x}\,\frac{1}{12}\mathbf{M}\,\mathbf{d}^2\boldsymbol{\omega}^2$$

$$= \left(\frac{1}{8}\mathbf{m}\,\mathbf{d}^2 + \frac{1}{24}\mathbf{M}\,\mathbf{d}^2\right)\boldsymbol{\omega}^2$$

$$= \left(1 + \frac{10}{3}\right)\text{x}\,(0.23)^2$$

Final Energy = 0.23 J.

Loss of Energy = $\mathbf{E_i} - \mathbf{E_f}$

= $1.0 - 0.23$

= 0.77 J. (or 0.8 J)

Alternate solution by not substituting in values until the end of the solution

(a) Angular momentum is conserved.

Angular Momentum Before = Angular Momentum After

$$\mathbf{L_i}(\text{Mass}) = \mathbf{L_f}(\text{Mass}) + \mathbf{L_f}(\text{Rod})$$

$$\frac{\mathbf{d}}{2}\mathbf{m}\,\mathbf{v} = \mathbf{m}\left(\frac{\mathbf{d}}{2}\right)^2 \boldsymbol{\omega} + \frac{1}{12}\mathbf{M}\,\mathbf{d}^2\boldsymbol{\omega}$$

$$\frac{\mathbf{d}}{2}\mathbf{m}\,\mathbf{v} = \left(\mathbf{m}\frac{\mathbf{d}^2}{4} + \frac{\mathbf{M}}{12}\mathbf{d}^2\right)\boldsymbol{\omega}$$

$$\mathbf{m}\,\mathbf{v} = \left(\frac{\mathbf{md}}{2} + \frac{\mathbf{Md}}{6}\right)\boldsymbol{\omega}$$

$$\boldsymbol{\omega} = \frac{6\ \mathbf{m\,v}}{\mathbf{M\,d} + 3\mathbf{m\,d}}$$

$$\boldsymbol{\omega} = \frac{6 \text{ x } 0.5 \text{ x } 2}{5 \text{ x } 20 + 3 \text{ x } 0.5 \text{ x } 0.4} = \frac{6}{26} = 0.23 \text{ rad/s}$$

(b) Initial Energy = $\frac{1}{2}\mathbf{m\,v}^2$

Final Energy $= \frac{1}{2}\mathbf{I}_{\text{Mass}}\,\boldsymbol{\omega}^2 + \frac{1}{2}\mathbf{I}_{\text{Rod}}\,\boldsymbol{\omega}^2$

$$= \frac{1}{2}\mathbf{m}\frac{\mathbf{d}^2}{4}\boldsymbol{\omega}^2 + \frac{1}{2} \text{ x } \frac{1}{12}\mathbf{M\ d}^2\boldsymbol{\omega}^2$$

$$= \left(\frac{1}{8}\mathbf{m\ d}^2 + \frac{1}{24}\mathbf{M\ d}^2\right)\boldsymbol{\omega}^2$$

$$= \left(\frac{1}{8}\mathbf{m\ d}^2 + \frac{1}{24}\mathbf{M\ d}^2\right)\frac{36\mathbf{m}^2\,\mathbf{v}^2}{(\mathbf{M\,d} + 3\mathbf{m\,d})^2}$$

$$= \frac{\mathbf{d}}{24}(3\mathbf{m\,d} + \mathbf{M\,d})\frac{36\mathbf{m}^2\,\mathbf{v}^2}{(\mathbf{M\,d} + 3\mathbf{m\,d})^2}$$

$$= \frac{3}{2}\frac{\mathbf{d\,m}^2\,\mathbf{v}^2}{(\mathbf{M\,d} + 3\mathbf{m\,d})}$$

Loss of Energy = $\mathbf{E_i} - \mathbf{E_f}$

$$= \frac{1}{2}\mathbf{m\,v}^2 - \frac{3}{2}\frac{\mathbf{d\,m}^2\,\mathbf{v}^2}{(\mathbf{M\,d} + 3\mathbf{m\,d})}$$

$$= \frac{1}{2}\mathbf{m\,v}^2\left(1 - \frac{3\mathbf{d\,m}}{\mathbf{M\,d} + 3\mathbf{m\,d}}\right)$$

$$= \frac{1}{2}\mathbf{m\,v}^2\left(\frac{\mathbf{M\,d} + 3\mathbf{d\,m} - 3\mathbf{d\,m}}{\mathbf{M\,d} + 3\mathbf{m\,d}}\right)$$

Loss of Energy = $\frac{1}{2}\mathbf{m\,v}^2\left(\frac{\mathbf{M\,d}}{\mathbf{M\,d} + 3\mathbf{m\,d}}\right) = \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } 4\left(\frac{5 \text{ x } 4}{5 \text{ x } 4 + 3 \text{ x } \frac{1}{2} \text{ x } 4}\right) = \frac{20}{26}$

= 0.77 J. (or 0.8 J)

Question 2

(a)

Define the direction to the right as positive.

The potential energy of $\mathbf{m_1}$ at **A** is converted to kinetic energy at **B** where $\mathbf{m_1}$ has a speed $\mathbf{v_{1i}}$ just before colliding with $\mathbf{m_2}$.

$$\mathbf{m_1\, g\, h} = \tfrac{1}{2}\mathbf{m_1\, v_{1i}^2}$$

$$9.8 \text{ x } 5.00 = \tfrac{1}{2}\ \mathbf{v_{1i}^2}$$

$$\mathbf{v_{1i}} = \sqrt{98} \ = 9.90 \text{ m/s.}$$

Call $\mathbf{v_{1f}}$ the speed of $\mathbf{m_1}$ at **B** just after the collision.

From your aid sheet you should have the formulae for collisions given in Serway on page 262 (one could always start from conservation of momentum and conservation of energy and solve for $\mathbf{v_{1f}}$).

$$\mathbf{v_{1f}} = \left(\frac{\mathbf{m_1 - m_2}}{\mathbf{m_1 + m_2}}\right)\mathbf{v_{1i}}$$

$$\mathbf{v_{1f}} = \left(\frac{5.00 - 10.0}{5.00 + 10.0}\right)9.90 = -\frac{1}{3}\text{ x } 9.90 \ = -3.30 \text{ m/s}$$

$\mathbf{m_1}$ is now moving to the left and as it rises up the track its kinetic energy will be converted back to potential energy as it until it reaches $\mathbf{h_{max}}$ where its speed will be zero.

$$\mathbf{m_1\, g\, h_{max}} = \tfrac{1}{2}\mathbf{m_1\, v_{1f}^2}$$

$$9.8 \text{ x } \mathbf{h_{max}} = \tfrac{1}{2}(-3.30)^2$$

$$\mathbf{h_{max}} = \tfrac{1}{2}\frac{10.89}{9.8} \ = 0.556 \text{ m.}$$

(b)

If the track were aluminum instead of wood, the moving magnet would induce eddy currents in the track which would create a magnetic field opposing the motion of $\mathbf{m_1}$ and not all the potential energy would be converted to kinetic energy resulting in a smaller $\mathbf{h_{max}}$.

Question 3

(a) $\mathbf{F}\text{ (N)} = \mathbf{K}\text{ (units)}\,\mathbf{m}\text{ (kg)}\,\mathbf{M}\text{ (kg)}\,\mathbf{e}^{-\mathbf{r}}$ (no units)

$$\mathbf{K}\text{ (units)} = \frac{\mathbf{F}\text{ (N)}}{\mathbf{m}\text{ (kg)}\,\mathbf{M}\text{ (kg)}}$$

Therefore the units of **K** are $\frac{\text{N}}{\text{kg}^2}$.

(b) $\mathbf{F_1} = \mathbf{K\,m\,M\,e}^{-\mathbf{R_1}}$ (1)

$\mathbf{F_2} = \mathbf{K\,m\,M\,e}^{-\mathbf{R_2}}$ (2)

$\mathbf{F_2} = \frac{1}{5}\mathbf{F_1}$ for $\mathbf{R_2} = 2\mathbf{R_1}$ and substituting into (2) gives

$\frac{1}{5}\mathbf{F_1} = \mathbf{K\,m\,M\,e}^{-2\mathbf{R_1}}$ (3) and dividing (1) by (3) gives

$$\frac{1}{\frac{1}{5}} = \frac{\mathbf{K\,m\,M\,e}^{-\mathbf{R_1}}}{\mathbf{K\,m\,M\,e}^{-2\mathbf{R_1}}}$$

$5 = \frac{\mathbf{e}^{-\mathbf{R_1}}}{\mathbf{e}^{-2\mathbf{R_1}}} = \mathbf{e}^{+\mathbf{R_1}}$ and taking logarithms of both sides of this equation gives

$\mathbf{R_1}$ = ln (5) = 1.61 (recall that there is a constant with dimensions and magnitude 1)

$\mathbf{R_1}$ = 1.61 m.

(c) If **M** is located at the origin and $\vec{\mathbf{r}}$ is the position vector for **m** then by definition

$\mathbf{U} = -\int_{\mathbf{r_{ref}}}^{\mathbf{r}} \vec{\mathbf{F}}\cdot\mathbf{d}\vec{\mathbf{r}}$ where $\mathbf{r_{ref}} = \infty$

Where $\vec{\mathbf{F}} = -\mathbf{K\,m\,M\,e}^{-\mathbf{r}}\,\hat{\mathbf{r}}$ and $\mathbf{d}\vec{\mathbf{r}} = \mathbf{dr}\,\hat{\mathbf{r}}$ (or $\vec{\mathbf{F}} = -\mathbf{K\,m\,M\,e}^{-\mathbf{x}}\,\hat{\mathbf{i}}$ and $\mathbf{d}\vec{\mathbf{r}} = \mathbf{dx}\,\hat{\mathbf{i}}$)

$$\mathbf{U} = -\int_{\infty}^{\mathbf{r}} (-\mathbf{K\,m\,M\,e}^{-\mathbf{r}}\,\hat{\mathbf{r}})\cdot\mathbf{dr}\,\hat{\mathbf{r}} = -\int_{\infty}^{\mathbf{r}} -\mathbf{K\,m\,M\,e}^{-\mathbf{r}}\,\mathbf{dr} = +\mathbf{K\,m\,M}\int_{\infty}^{\mathbf{r}} \mathbf{e}^{-\mathbf{r}}\,\mathbf{dr}$$

The derivative of $\mathbf{e}^{-\mathbf{r}}$ is $-\mathbf{e}^{-\mathbf{r}}$ (the derivative of $-\mathbf{e}^{-\mathbf{r}}$ is $\mathbf{e}^{-\mathbf{r}}$) so the integral of $\mathbf{e}^{-\mathbf{r}}$ is $-\mathbf{e}^{-\mathbf{r}}$.

$$\mathbf{U} = \mathbf{K\,m\,M}\int_{\infty}^{\mathbf{r}} \mathbf{e}^{-\mathbf{r}}\,\mathbf{dr} = \mathbf{K\,m\,M}\left[-\mathbf{e}^{-\mathbf{r}}\right]\Big|_{\infty}^{\mathbf{r}} = -\mathbf{K\,m\,M\,e}^{-\mathbf{r}}$$

$$\mathbf{U} = -\mathbf{K\,m\,M\,e}^{-\mathbf{r}}$$

(d) Yes

(e) The exponent in $\mathbf{e}^{-\mathbf{r}}$ can't have any units so there must be a constant **k** in front of **r** which has a magnitude of 1 but units of m^{-1}. The integral of $\mathbf{e}^{-\mathbf{kr}}$ is $-\frac{1}{\mathbf{k}}\mathbf{e}^{-\mathbf{kr}}$. Then for determining units $\mathbf{U} = -\frac{\mathbf{K\,m\,M\,e}^{-\mathbf{r}}}{\mathbf{k}}$

$$\mathbf{U}\text{ (units)} = \frac{\mathbf{K}\left(\frac{\text{N}}{\text{kg}^2}\right)\mathbf{m}\text{ (kg)}\,\mathbf{M}\text{ (kg)}}{\mathbf{k}\,(\text{m}^{-1})} = \text{N m} = \text{J}$$