Solutions to Questions of Term Test I 2005

Problem 1

(a) $\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$ and since $\vec{\mathbf{r}}$ and $\vec{\mathbf{F}}$ are perpendicular

$\boldsymbol{\tau} = \mathbf{rF} =$ 30.0 x 0.800 = 24.0 N·m

(b) $\boldsymbol{\tau} = \mathbf{I\alpha}$

$\boldsymbol{\alpha} =$ 24.0 /(0750 x 30.0 x 30.0) = 3.56 x 10^{-2} s^{-2} (rad/s^2)

This question is marked for correct units.

(c) $\mathbf{a_t} = \boldsymbol{\alpha}\ \mathbf{r} =$ 3.56 x 10^{-2} x 30.0 = 1.07 m/s^2

(d) $\mathbf{K} = \frac{1}{2}\,\mathbf{mv^2}$ and when $\mathbf{K}$ = 50.0 J

$$\mathbf{v} = \sqrt{\frac{2 \text{ x } 50.0}{0.750}} = 11.547$$

$$\mathbf{a_r} = \frac{\mathbf{v^2}}{\mathbf{r}} = \frac{11.547^2}{30.0} = 4.444 \text{ m/s}^2$$

$$\vec{\mathbf{a}} = \vec{\mathbf{a}}_{\mathbf{r}} + \vec{\mathbf{a}}_{\mathbf{t}}$$

$$\mathbf{a} = \sqrt{\mathbf{a_t^2} + \mathbf{a_r^2}} = \sqrt{1.068^2 + 4.444^2} = \sqrt{20.890} = 4.57 \text{ m/s}^2$$

This question is marked for the correct number of significant figures.

Problem 2

(a) $\mathbf{k} = \frac{\mathbf{F}}{\mathbf{x}} = \frac{10.0}{0.500}$ N/m

$\mathbf{T} = \mathbf{2}\,\boldsymbol{\pi}\sqrt{\frac{\mathbf{m}}{\mathbf{k}}} = 2\boldsymbol{\pi}\sqrt{\frac{1.00}{200}}$ = 0.444 s

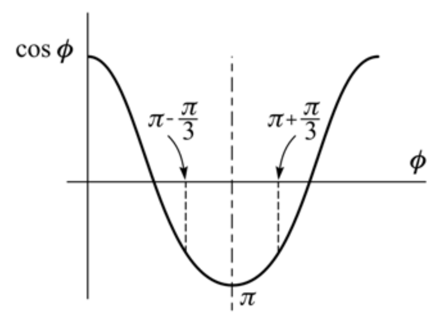
(b) **x = A cos(ωt)** describes the motion since **x = A** when **t** = 0 where **A** = 5.00 cm.

$\boldsymbol{\omega} = \sqrt{\frac{\mathbf{k}}{\mathbf{m}}} = \sqrt{\frac{200}{1.00}} = \sqrt{200}$ = 14.14 s^{-1}

At **t** = 0.200 s,

x = 5.00 **cos** (14.14 x 0.200) = 5.00 x (- 0.951) = -4.76 cm

(c) $\mathbf{x} = \mathbf{A}\,\mathbf{cos}(\boldsymbol{\omega}\,\mathbf{t} + \varphi)$ where **x** = - ½ **A** at **t** = 0.



- ½ **A** = $\mathbf{A}\,\mathbf{cos}(\varphi)$

$\mathbf{cos}(\varphi)$ = - ½

Referring to the diagram, φ is in the second or third quadrant.

$\varphi = \boldsymbol{\pi} + \frac{\boldsymbol{\pi}}{3} = \frac{4}{3}\boldsymbol{\pi}$ (4.19) or

$\varphi = \boldsymbol{\pi} - \frac{\boldsymbol{\pi}}{3} = \frac{2}{3}\boldsymbol{\pi}$ (2.09)

But it is moving to the right so the slope of the graph must be positive and so $\varphi = \frac{4}{3}\boldsymbol{\pi}$

$\mathbf{x} = 5.00\,\mathbf{cos}(14.1\,\mathbf{t} + \frac{4}{3}\boldsymbol{\pi})$

$\mathbf{x} = 5.00\,\mathbf{cos}(14.1\,\mathbf{t} - \frac{2}{3}\boldsymbol{\pi})$ is also correct and part marks is given for

$\mathbf{x} = 5.00\,\mathbf{cos}(14.1\,\mathbf{t} + \frac{2}{3}\boldsymbol{\pi})$

(d) When the mass moves, one of the springs is compressed and the other is stretched. When one of the springs is removed then only half the force is required to move the mass the same distance and thus the spring constant of the “ new” spring is half the original spring constant.

$\mathbf{T}' = 2\boldsymbol{\pi}\sqrt{\frac{\mathbf{m}}{\mathbf{k}'}}$ where $\mathbf{k}' = \frac{1}{2}\mathbf{k}$

$\mathbf{T}' = 2\boldsymbol{\pi}\sqrt{\frac{\mathbf{2\,m}}{\mathbf{k}}} = \sqrt{\mathbf{2}}\ \mathbf{T}$ which is a larger period.

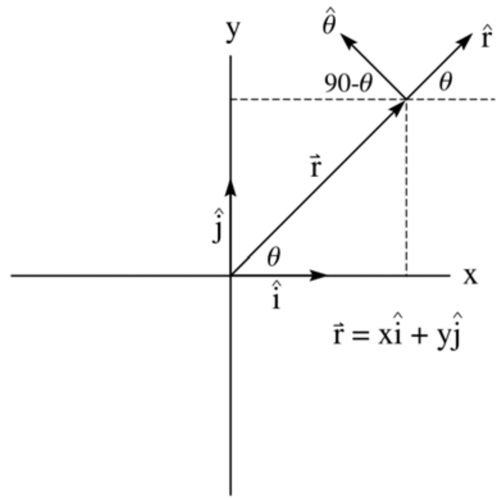
Problem 3

The position vector for an arbitrary point in cylindrical coordinates is $\vec{\mathbf{r}} = \mathbf{r}\,\hat{\mathbf{r}}$.

$$\frac{\mathbf{d}\vec{\mathbf{r}}}{\mathbf{dt}} = \frac{\mathbf{d}}{\mathbf{dt}}\left(\mathbf{r}\,\hat{\mathbf{r}}\right) = \frac{\mathbf{dr}}{\mathbf{dt}}\hat{\mathbf{r}} + \mathbf{r}\frac{\mathbf{d}\hat{\mathbf{r}}}{\mathbf{dt}}$$

Since the particle is in uniform circular motion then **r** is constant and $\frac{\mathbf{dr}}{\mathbf{dt}} = \mathbf{0}$.

Therefore $\frac{\mathbf{d}\vec{\mathbf{r}}}{\mathbf{dt}} = \mathbf{r}\frac{\mathbf{d}\hat{\mathbf{r}}}{\mathbf{dt}}$ (1). From the diagram



$\hat{\boldsymbol{\theta}} = -\mathbf{cos(90-\boldsymbol{\theta})}\,\hat{\mathbf{i}} + \mathbf{cos}\boldsymbol{\theta}\,\hat{\mathbf{j}} = -\mathbf{sin}\boldsymbol{\theta}\,\hat{\mathbf{i}} + \mathbf{cos}\boldsymbol{\theta}\,\hat{\mathbf{j}}$ (2) and

$\hat{\mathbf{r}} = \mathbf{cos}\boldsymbol{\theta}\,\hat{\mathbf{i}} + \mathbf{sin}\boldsymbol{\theta}\,\hat{\mathbf{j}}$ and therefore

$$\frac{\mathbf{d}\hat{\mathbf{r}}}{\mathbf{dt}} = -\mathbf{sin}\boldsymbol{\theta}\frac{\mathbf{d}\boldsymbol{\theta}}{\mathbf{dt}}\hat{\mathbf{i}} + \mathbf{cos}\boldsymbol{\theta}\frac{\mathbf{d}\boldsymbol{\theta}}{\mathbf{dt}}\hat{\mathbf{j}} = \frac{\mathbf{d}\boldsymbol{\theta}}{\mathbf{dt}}(-\mathbf{sin}\boldsymbol{\theta}\,\hat{\mathbf{i}} + \mathbf{cos}\boldsymbol{\theta}\,\hat{\mathbf{j}}) \qquad (3)$$

Substituting (2) into (3) gives $\frac{\mathbf{d}\hat{\mathbf{r}}}{\mathbf{dt}} = \frac{\mathbf{d}\boldsymbol{\theta}}{\mathbf{dt}}\hat{\boldsymbol{\theta}}$ (4)

Substituting (3) into (1) gives $\frac{\mathbf{d}\vec{\mathbf{r}}}{\mathbf{dt}} = \mathbf{r}\frac{\mathbf{d}\boldsymbol{\theta}}{\mathbf{dt}}\hat{\boldsymbol{\theta}}$ (5)

Since the particle is in uniform circular motion the value of $\frac{\mathbf{d}\boldsymbol{\theta}}{\mathbf{dt}}$ can be obtained considering one complete revolution.

$\frac{\mathbf{d}\boldsymbol{\theta}}{\mathbf{dt}} = \frac{\mathbf{2}\boldsymbol{\pi}}{\mathbf{T}}$ where the period of the motion is obtained from the circumference of the circle with radius **r** and the speed **v** of the particle. Since $\mathbf{d} = \mathbf{v}\,\mathbf{t}$

$\mathbf{2}\boldsymbol{\pi}\mathbf{r} = \mathbf{v}\,\mathbf{T}$ or $\mathbf{T} = \frac{\mathbf{2}\boldsymbol{\pi}\mathbf{r}}{\mathbf{v}}$ and therefore $\frac{\mathbf{d}\boldsymbol{\theta}}{\mathbf{dt}} = \frac{\mathbf{2}\boldsymbol{\pi}}{\frac{\mathbf{2}\boldsymbol{\pi}\mathbf{r}}{\mathbf{v}}} = \frac{\mathbf{v}}{\mathbf{r}}$ (6).

Alternatively one can get $\frac{\mathbf{d}\boldsymbol{\theta}}{\mathbf{dt}}$ by considering the definition of $\boldsymbol{\theta}$ in radians. $\boldsymbol{\theta} = \frac{\mathbf{s}}{\mathbf{r}}$ where **s** is the arc length and **r** is the radius of the circular path of the particle which is constant.

$$\frac{\mathbf{d}\boldsymbol{\theta}}{\mathbf{dt}} = \frac{\mathbf{d}}{\mathbf{dt}}\left(\frac{\mathbf{s}}{\mathbf{r}}\right) = \frac{\mathbf{1}}{\mathbf{r}}\frac{\mathbf{ds}}{\mathbf{dt}} = \frac{\mathbf{1}}{\mathbf{r}}\mathbf{v} = \frac{\mathbf{v}}{\mathbf{r}} \qquad (6)$$

Substituting into (6) into (5) gives $\frac{\mathbf{d}\vec{\mathbf{r}}}{\mathbf{dt}} = \mathbf{r}\frac{\mathbf{v}}{\mathbf{r}}\hat{\boldsymbol{\theta}} = \mathbf{v}\,\hat{\boldsymbol{\theta}} = \vec{\mathbf{v}}$